ECE343 Spring 2025

Objective 8.1: Displacement Current



SCHEDULE

Lesson	Topics	Reading	Notes
24	BVP (Capacitor, Coax), Electric Potential	5.16	LO 5.3
25	Electric Boundary Conditions	5.18-5.22	LO 5.4
26	BC's/Capacitance	5.23-5.24	
27	Electrostatic Lab	Lab packet	
Chapter 6: Steady Current & Conductivity			
28	Current density, Conductivity, Resistance	Ch. 6	
Chapter 7: Magnetostatics			
29	Magnetic Fields, Gauss' Law for Magnetism	7.1-7.3	LO 7.1-7.2 **GR 2**
30	Ampere's Law	7.4-7.5, 7.9	LO 7.3
31	Ampere's Law Applied	7.6-7.8	LO 7.3
32	Magnetic Boundary Conditions	7.10-7.14	LO 7.4
33	Magnetostatics Lab	Lab packet	
Chapter 8: Time-Varying Fields			
34	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.1
35	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.2-8.3
Chapter 9: Plane Waves in Lossless Media			
36	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
37	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
38	Uniform Plane Waves & Polarization	9.4-9.6	LO 9.2-9.3
39	Normal Incidence	Vol 2, 5.6-5.7	LO 9.4-9.5
40	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
41	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
42	Incidence Lab	TBD	LO 9.5

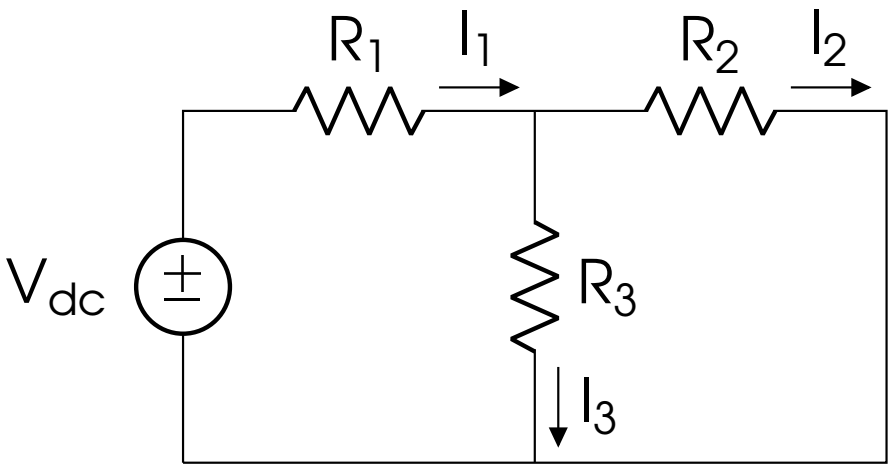
Ch 8 Learning Objectives

7.1 I can articulate the significance of the displacement current and calculate it, given a time-varying electric field.

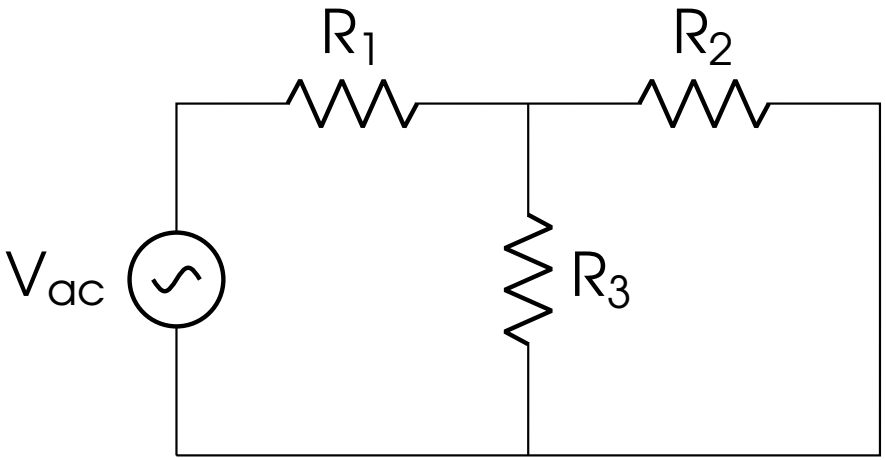
7.2 I can articulate the meaning of and calculate the relaxation time for a variety of materials.

7.3 I know the forms of Maxwell's equations for time-varying fields and use them to calculate electric and magnetic fields for a variety of physical systems.

Displacement Current



KVL: $I_1 - I_2 - I_3 = 0$



$$I = -\frac{\partial Q}{\partial t}$$

$$\int \vec{J} \cdot \vec{dS} = -\frac{\partial}{\partial t} \iiint \rho_v \, dV$$

$$\iiint \left(\nabla \cdot \vec{J} \right) dV = \iiint \left(-\frac{\partial}{\partial t} \right) \rho_v \, dV$$

$$\Longrightarrow \quad \nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

Insight #1: Relaxation Time

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

Ohm's Law $\longrightarrow \vec{J} = \sigma \vec{E}$

$$\implies \nabla \cdot \sigma \vec{E} = -\frac{\partial \rho_v}{\partial t}$$

$$\implies \nabla \cdot \vec{E} = -\frac{1}{\sigma}\frac{\partial \rho_v}{\partial t}$$

...but remember Gauss?

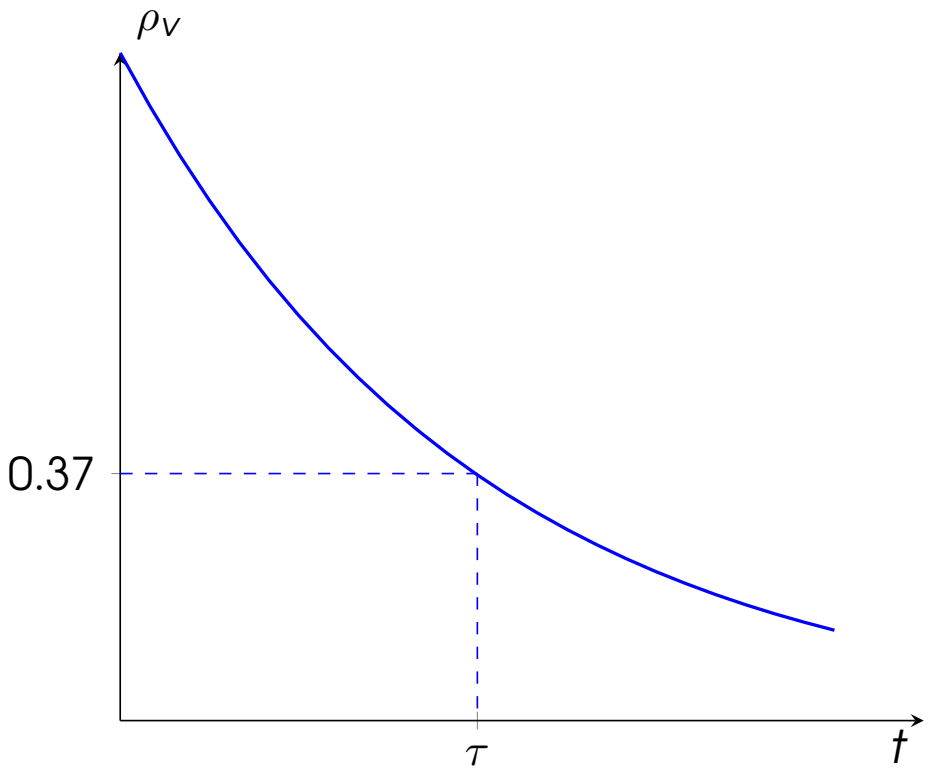
$$\implies \nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon}$$

$$\implies \frac{\rho_v}{\varepsilon} + \frac{1}{\sigma}\frac{\partial \rho_v}{\partial t} = 0$$

$$\implies \rho_v = \rho_0 e^{-t/\tau}$$

$$\implies -\frac{1}{\tau} + \frac{\sigma}{\varepsilon} = 0 \implies \tau = \frac{\varepsilon}{\sigma}$$

$$\boxed{\rho_v = \rho_0 e^{-t\frac{\sigma}{\varepsilon}}}$$



Relaxation Time: Examples





Polystyrene

$$\sigma = 10^{-17}\text{S/m and } \varepsilon = 2.56\varepsilon_0$$

$$\tau = \frac{2.56\varepsilon_0\ \text{F/m}}{10^{-17}\ \text{S/m}} = 2.3 \times 10^6\text{s}$$

$$\approx 26\ \text{days}$$

Copper

$$\sigma = 5.8 \times 10^7\text{S/m and } \varepsilon = \varepsilon_0$$

$$\tau = \frac{\varepsilon_0\text{F/m}}{5.8 \times 10^7\text{S/m}} = 1.5 \times 10^{-19}\text{s}$$

Insight #2: Ampere's Law

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

Ampere's Law $\longrightarrow \nabla \times \vec{H} = \vec{J}$

$$\nabla \cdot \nabla \times \vec{H} = \nabla \cdot \vec{J}$$

$$0 = -\frac{\partial \rho_v}{\partial t} \stackrel{?}{=} \nabla \cdot \vec{J}$$

Maxwell to the rescue!

$$\implies \vec{J} = \underbrace{\vec{J}_c}_{\text{conduction}} + \underbrace{\vec{J}_d}_{\text{displacement}}$$

$$\implies \nabla \cdot \vec{J} = \nabla \cdot \vec{J}_c + \nabla \cdot \vec{J}_d = 0$$

$$\implies \nabla \cdot \vec{J}_c = -\nabla \cdot \vec{J}_d = -\frac{\partial \rho_v}{\partial t}$$

$$\implies \nabla \cdot \vec{J}_d = \frac{\partial \rho_v}{\partial t}$$

Use Gauss' Law, $(\nabla \cdot \vec{E} = \frac{\rho_v}{\varepsilon})$

$$\nabla \cdot \vec{J}_d = \frac{\partial}{\partial t} \nabla \cdot \varepsilon \vec{E}$$

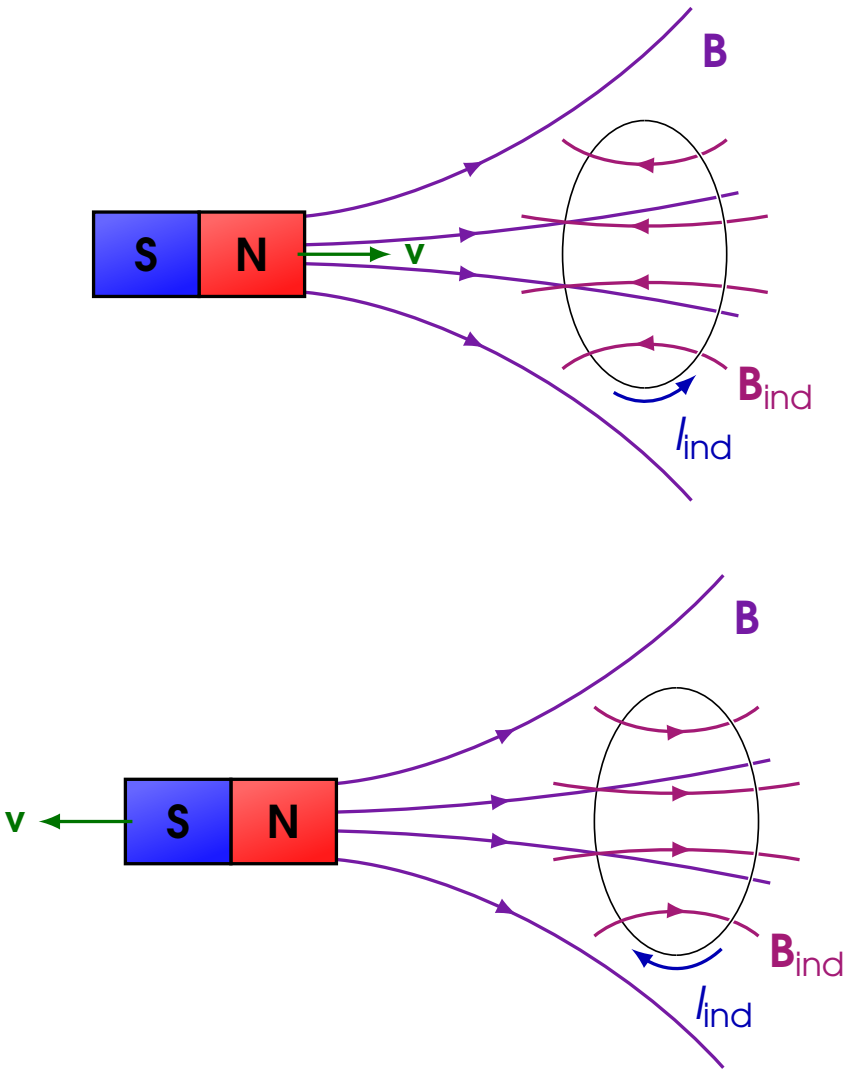
$$\vec{J} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

We know $\vec{J}_c = \sigma \vec{E}$, so...

Ampere's Law

$$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_d = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$

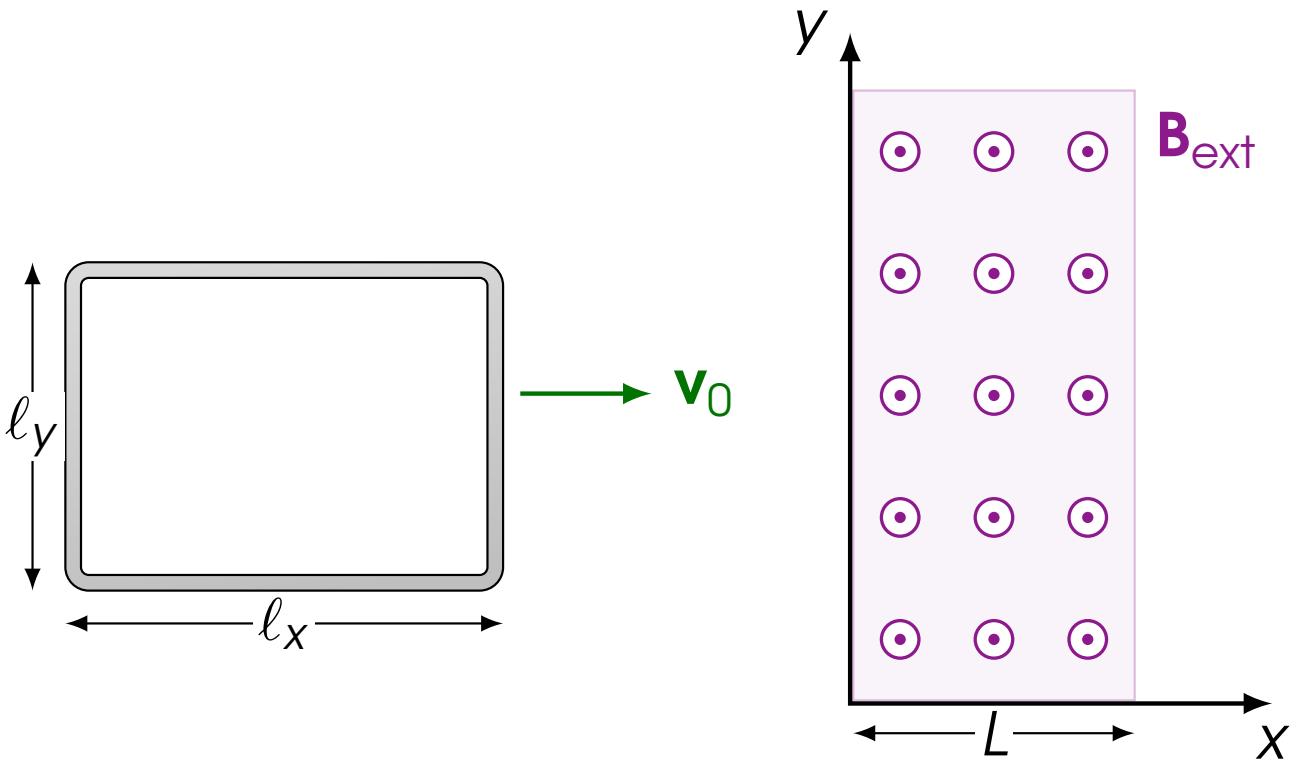
Lenz's Law



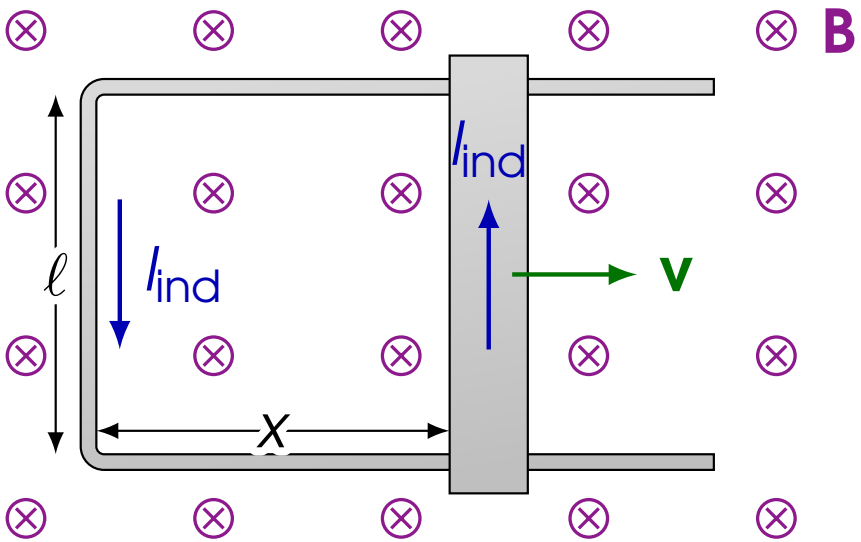
Lenz's Law

A current is induced by a change in the impressed magnetic field. The induced current results from an induced magnetic field opposed to the impressed magnetic field.

Lenz's Law



Lenz's Law



Faraday's Law

Faraday's Law is the mathematical form of Lenz's Law:

$$V = -\frac{\partial \Psi_m}{\partial t} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot d\vec{S}$$

Electric potential comes from the electric field:

$$V = \int \vec{E}\cdot d\vec{\ell}$$

$$\int \vec{E}\cdot d\vec{\ell} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot d\vec{S}$$

$$\iint \left(\nabla \times \vec{E}\right)\cdot d\vec{S} = -\frac{\partial}{\partial t}\iint \vec{B}\cdot d\vec{S}$$

Faraday's Law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$