ECE343 Spring 2025

Objective 7.4: Magnetic Field Boundary Conditions



UNITED STATES AIR FORCE ACADEMY

Lesson	Topics	Reading	Notes
24	BVP (Capacitor, Coax), Electric Potential	5.16	LO 5.3
25	Electric Boundary Conditions	5.18-5.22	LO 5.4
26	BC's/Capacitance	5.23-5.24	
27	Electrostatic Lab	Lab packet	
Chapter 6: Steady Current & Conductivity			
28	Current density, Conductivity, Resistance	Ch. 6	
Chapter 7: Magnetostatics			
29	Magnetic Fields, Gauss' Law for Magnetism	7.1-7.3	LO 7.1-7.2 **GR 2**
30	Ampere's Law	7.4-7.5, 7.9	LO 7.3
31	Ampere's Law Applied	7.6-7.8	LO 7.3
32	Magnetic Boundary Conditions	7.10-7.14	LO 7.4
33	Magnetostatics Lab	Lab packet	
Chapter 8: Time-Varying Fields			
34	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.1
35	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.2-8.3
Chapter 9: Plane Waves in Lossless Media			
36	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
37	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
38	Uniform Plane Waves & Polarization	9.4-9.6	LO 9.2-9.3
39	Normal Incidence	Vol 2, 5.6-5.7	LO 9.4-9.5
40	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
41	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
42	Incidence Lab	TBD	LO 9.5

Ch 7 Learning Objectives

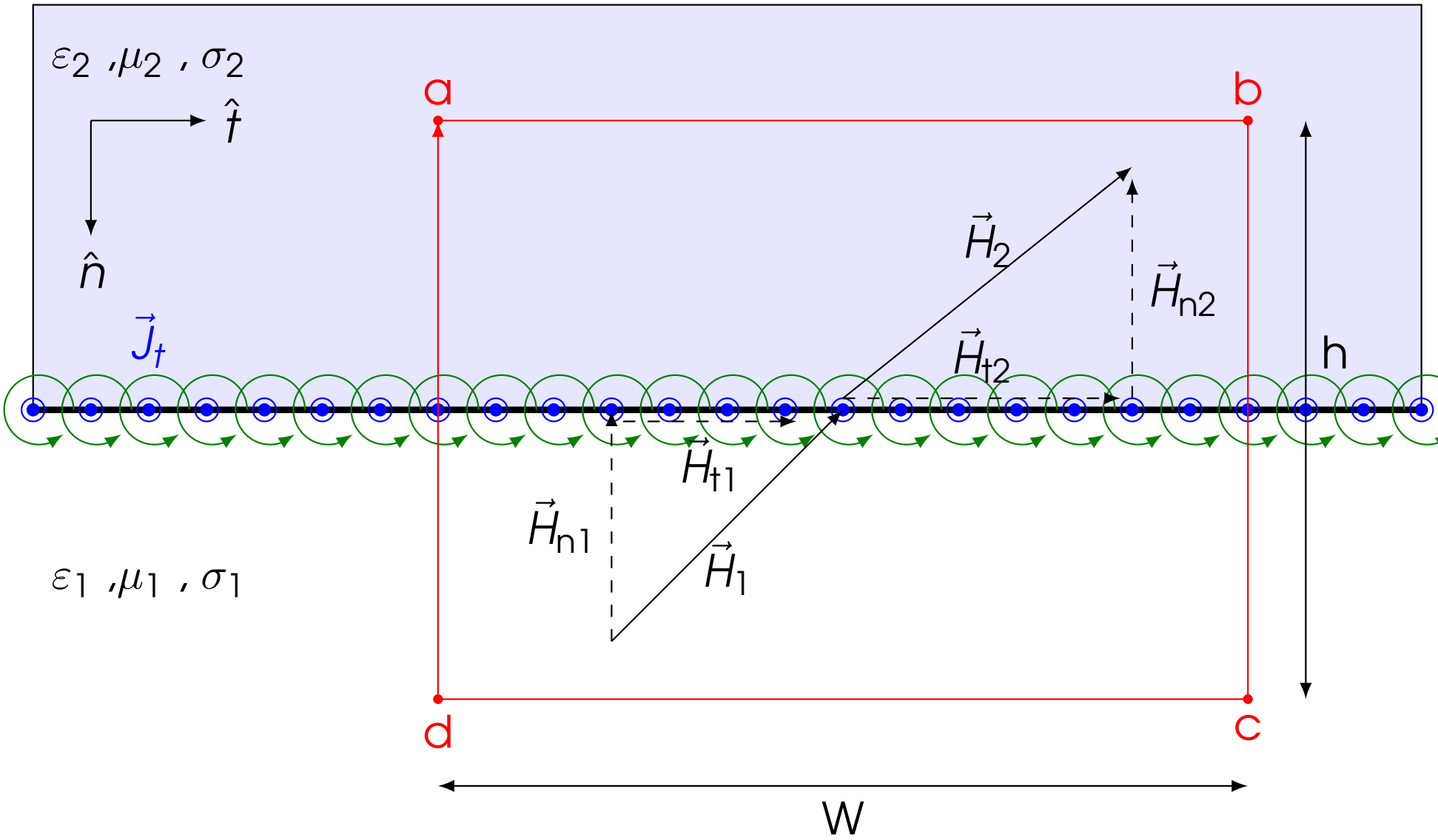
7.1 I can use the Biot-Savart law to simple magnetostatic systems to find the magnetic field at an observation point.

7.2 I can articulate the meaning of both the integral form and point form of Gauss' Law for magnetism.

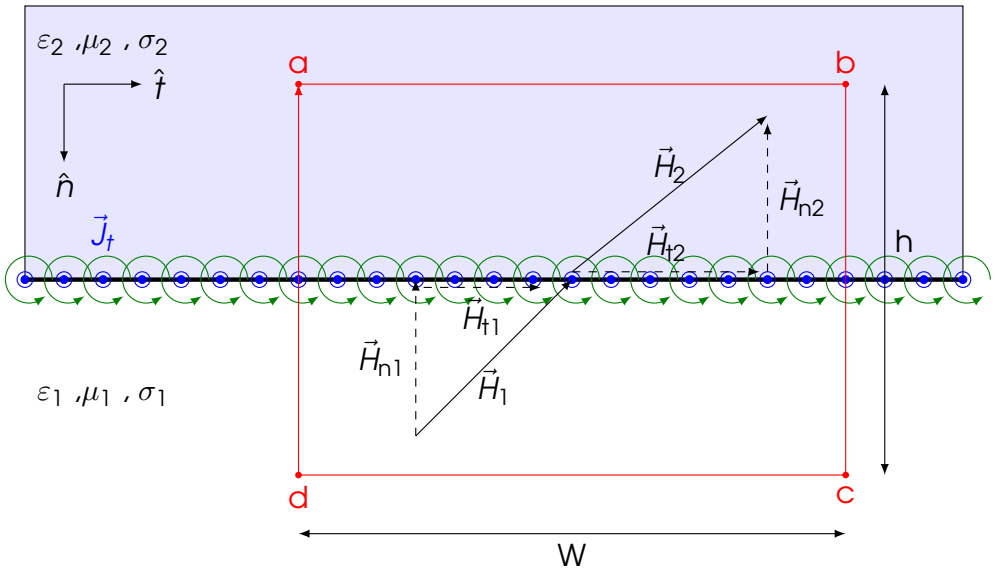
7.3 I can use Ampere's Law to calculate the magnetic field at an observation point for simple magnetostatic systems.

7.4 I can apply the magnetic field boundary conditions to calculate magnetic fields on both sides of a material boundary.

Magnetic Boundary Conditions



$\vec{H}$ Boundary Conditions: Transverse



Using Ampere's Law for magnetism:

$$\oint_C \vec{H} \cdot d\vec{\ell} = I_{encl}$$

We can write:

$$\textcircled{1} \quad \oint_C \vec{H} \cdot d\vec{\ell} = \int_a^d \vec{H} \cdot d\vec{\ell}_1 + \int_d^c \vec{H} \cdot d\vec{\ell}_2 + \int_c^b \vec{H} \cdot d\vec{\ell}_3 + \int_b^a \vec{H} \cdot d\vec{\ell}_4$$

$$= 0 + \int_d^c \vec{H} \cdot d\vec{\ell}_2 + 0 + \int_b^a \vec{H} \cdot d\vec{\ell}_4$$

$$= \int_d^c \vec{H}_1 \cdot \left(\hat{t} d\ell\right) + \int_a^b \vec{H}_2 \cdot \left(\hat{t} d\ell\right)$$

$$= \vec{H}_{1t} W - \vec{H}_{2t} W$$

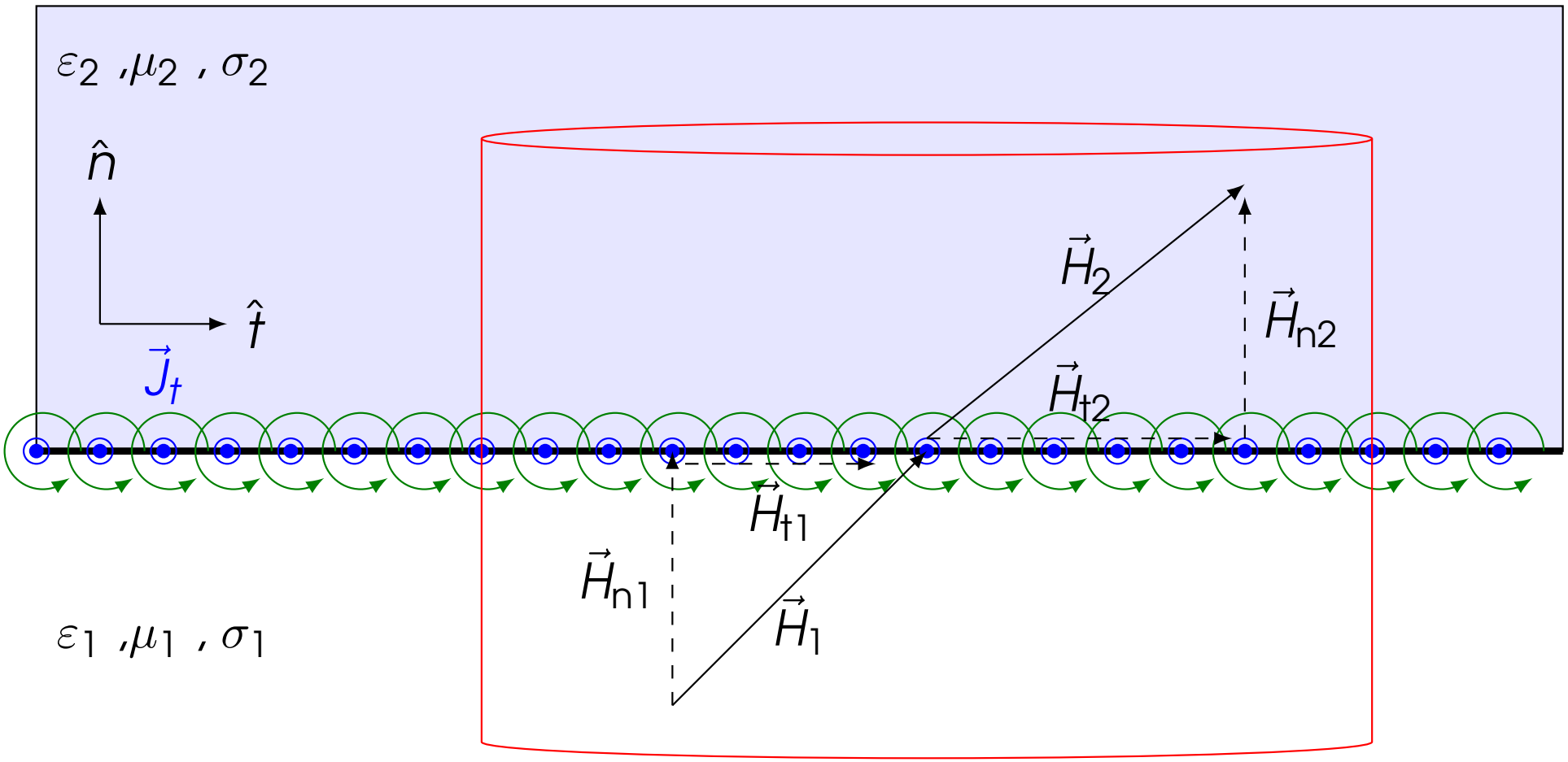
$$\textcircled{2} \quad I_{encls} = \vec{J}_t W$$

$$\Longrightarrow \oint_C \vec{H} \cdot d\vec{\ell} = I_{encl}$$

$$\Longrightarrow \vec{H}_{1t} W - \vec{H}_{2t} W = \vec{J}_t W$$

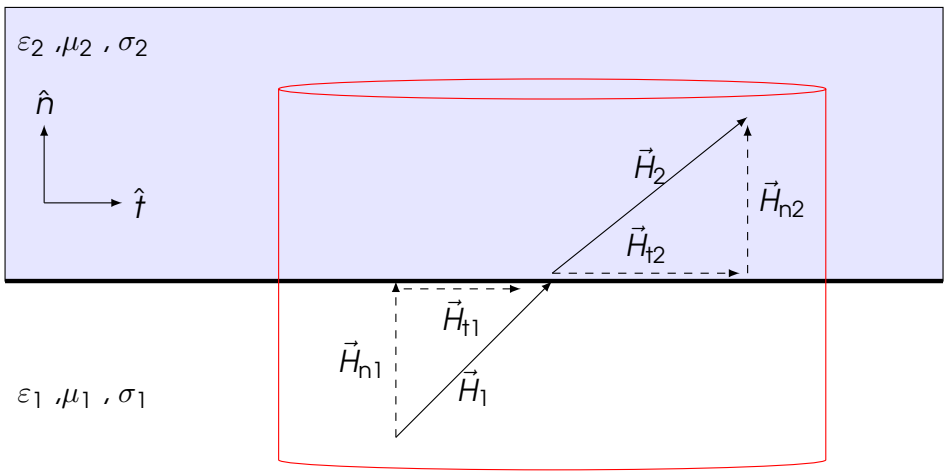
$$\Longrightarrow \hat{n}_{21} \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}$$

$\vec{H}$ Boundary Conditions: Normal



Gauss' Law $\Longrightarrow$ $\iint \vec{B} \cdot \vec{dS} = 0$

$\vec{H}$ Boundary Conditions: Normal



$$\iint \vec{B} \cdot d\vec{S} = 0$$

$$\iint_{\text{top}} \vec{B} \cdot d\vec{S} + \iint_{\text{bottom}} \vec{B} \cdot d\vec{S} + \iint_{\text{side}} \vec{B} \cdot d\vec{S} = 0$$

$$\lim_{h \longrightarrow 0} \Longrightarrow$$

$$\iint_{\text{top}} \vec{B} \cdot (\hat{n} dS) + \iint_{\text{bottom}} \vec{B} \cdot (-\hat{n} dS) + 0 = 0$$

$$\Longrightarrow \vec{B}_{n2}\,(\text{Area}) - \vec{B}_{n1}\,(\text{Area}) = 0$$

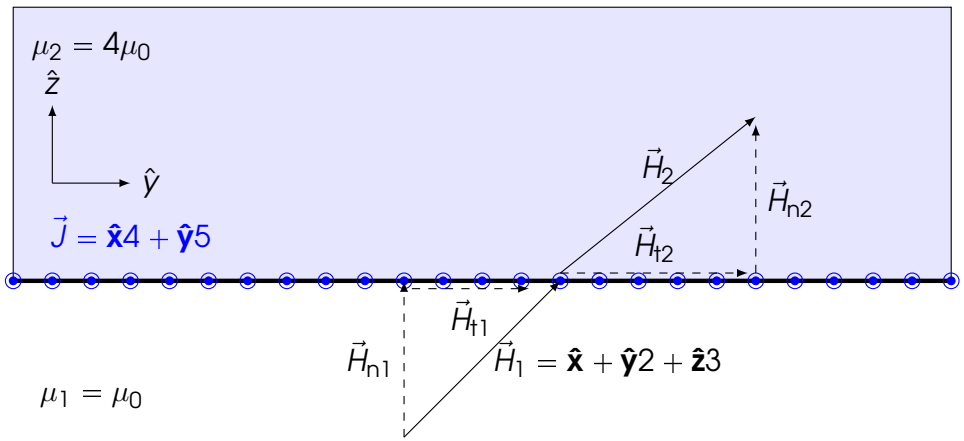
$$\Longrightarrow \mu_2 \vec{H}_{n2} = \mu_1 \vec{H}_{n1}$$

$\vec{H}$ Boundary Condition Summary

	Electric	**Magnetic**
Normal	$\hat{n}_{21} \cdot \left(\varepsilon_1 \vec{E}_1 - \varepsilon_2 \vec{E}_2 \right) = \rho_s$	$\mu_2 \vec{H}_{n2} = \mu_1 \vec{H}_{n1}$
Transverse	$\vec{E}_{t1} = \vec{E}_{t2}$	$\hat{n}_{21} \times \left(\vec{H}_1 - \vec{H}_2 \right) = \vec{J}$

Boundary Condition: Example

A magnetic field $\vec{H} = \hat{\mathbf{x}} + \hat{\mathbf{y}}2 + \hat{\mathbf{z}}3$ exists in a material with $\mu = \mu_0$ and is incident on a boundary with a surface current of $\vec{J} = \hat{\mathbf{x}}4 + \hat{\mathbf{y}}5$. The material on the other side of the boundary has $\mu = 4\mu_0$. Determine $\vec{H}_2$.



1. Draw a picture
2. Identify T & N

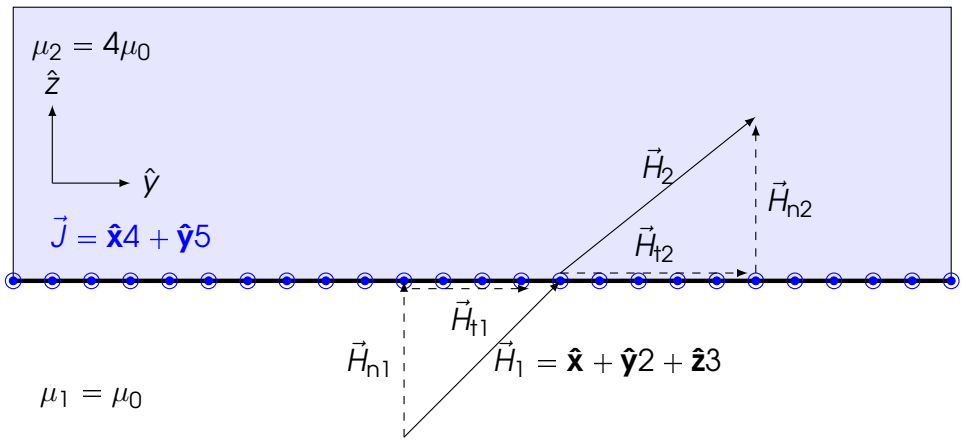
$$\vec{H}_{t1} = \hat{\mathbf{x}}1 + \hat{\mathbf{y}}2 \implies |\vec{H}_{t1}| = \sqrt{5}$$
$$\vec{H}_{n1} = \hat{\mathbf{y}}3 \implies |\vec{H}_{n1}| = 3$$

3. $\hat{n}_{2,1} = -\hat{\mathbf{z}}$
4. Normal BC

$$\hat{n}_{21} \times \left(\vec{H}_1 - \vec{H}_2\right) = \vec{J}$$
$$-\hat{\mathbf{z}} \times (\hat{\mathbf{x}} + \hat{\mathbf{y}}2 + \hat{\mathbf{z}}3)$$
$$-(-\hat{\mathbf{z}}) \times \left(\hat{\mathbf{x}}H_{2x} + \hat{\mathbf{y}}H_{2y} + \hat{\mathbf{z}}H_{2z}\right) = \vec{J}$$
$$-\hat{\mathbf{y}} + \hat{\mathbf{x}}2 + \hat{\mathbf{y}}H_{2x} - \hat{\mathbf{x}}H_{2y} = \hat{\mathbf{x}}4 + \hat{\mathbf{y}}5$$
$$\hat{\mathbf{x}}\left(2 - H_{2y}\right) + \hat{\mathbf{y}}\left(H_{2x} - 1\right) = \hat{\mathbf{x}}4 + \hat{\mathbf{y}}5$$
$$\implies H_{2y} = -2$$
$$\implies H_{2x} = 6$$

Boundary Condition: Example

A magnetic field $\vec{H} = \hat{\mathbf{x}} + \hat{\mathbf{y}}2 + \hat{\mathbf{z}}3$ exists in a material with $\mu = \mu_0$ and is incident on a boundary with a surface current of $\vec{J} = \hat{\mathbf{x}}4 + \hat{\mathbf{y}}5$. The material on the other side of the boundary has $\mu = 4\mu_0$. Determine $\vec{H}_2$.



(5) Normal BC

$$\mu_1 H_{n1} = \mu_2 H_{n2}$$

$$H_{2z} = \frac{\mu_0}{4\mu_0} 3 = \frac{3}{4}$$

(6) Put it all together

$$\vec{H}_2 = \hat{\mathbf{x}}6 - \hat{\mathbf{y}}2 + \hat{\mathbf{z}}\frac{3}{4}$$