ECE343 Spring 2025

Objective 7.3: Ampere's Law



UNITED STATES AIR FORCE ACADEMY

Lesson	Topics	Reading	Notes
24	BVP (Capacitor, Coax), Electric Potential	5.16	LO 5.3
25	Electric Boundary Conditions	5.18-5.22	LO 5.4
26	BC's/Capacitance	5.23-5.24	
27	Electrostatic Lab	Lab packet	
Chapter 6: Steady Current & Conductivity			
28	Current density, Conductivity, Resistance	Ch. 6	
Chapter 7: Magnetostatics			
29	Magnetic Fields, Gauss' Law for Magnetism	7.1-7.3	LO 7.1-7.2 **GR 2**
30	Ampere's Law	7.4-7.5, 7.9	LO 7.3
31	Ampere's Law Applied	7.6-7.8	LO 7.3
32	Magnetic Boundary Conditions	7.10-7.14	LO 7.4
33	Magnetostatics Lab	Lab packet	
Chapter 8: Time-Varying Fields			
34	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.1
35	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.2-8.3
Chapter 9: Plane Waves in Lossless Media			
36	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
37	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
38	Uniform Plane Waves & Polarization	9.4-9.6	LO 9.2-9.3
39	Normal Incidence	Vol 2, 5.6-5.7	LO 9.4-9.5
40	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
41	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
42	Incidence Lab	TBD	LO 9.5

Ch 7 Learning Objectives

7.1 I can use the Biot-Savart law to simple magnetostatic systems to find the magnetic field at an observation point.

7.2 I can articulate the meaning of both the integral form and point form of Gauss' Law for magnetism.

7.3 I can use Ampere's Law to calculate the magnetic field at an observation point for simple magnetostatic systems.

7.4 I can apply the magnetic field boundary conditions to calculate magnetic fields on both sides of a material boundary.

Ampere's Law

Integral Form

$$\oint_C \vec{H} \cdot d\ell = I_{\text{encl}}$$

$\Longrightarrow$ Relates the magnetic field along a closed *Amperian path* to the total current flowing through any surface bounded by that path.

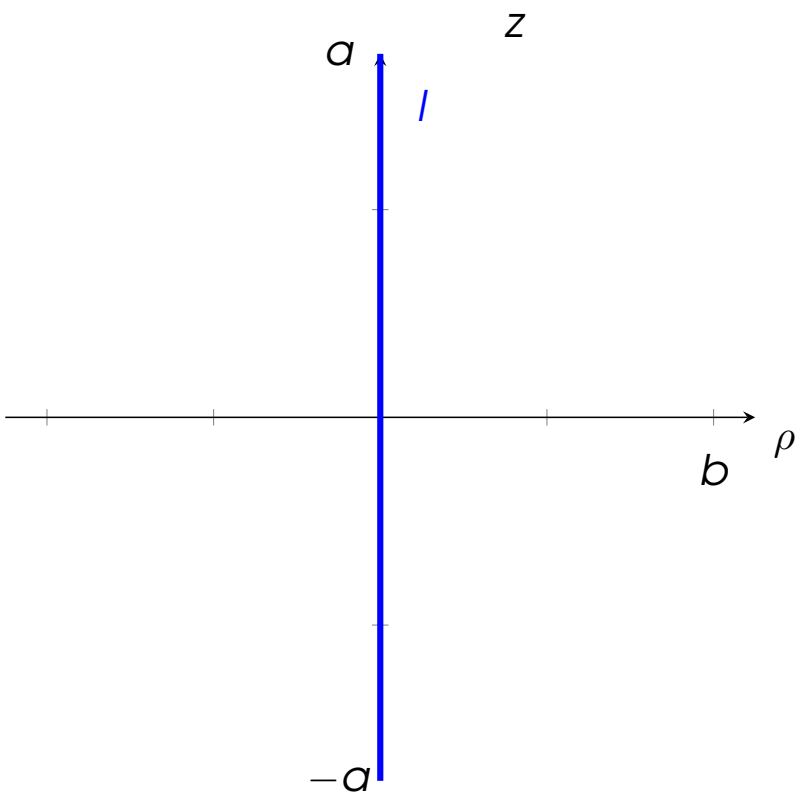
Point Form

$$\nabla \times \vec{H} = \vec{J}$$

$\Longrightarrow$ A current density produces a curling magnetic field.

Ampere's Law: Line Charge

Find $\vec{H}$ at a distance b from the LC.



1. Draw a picture

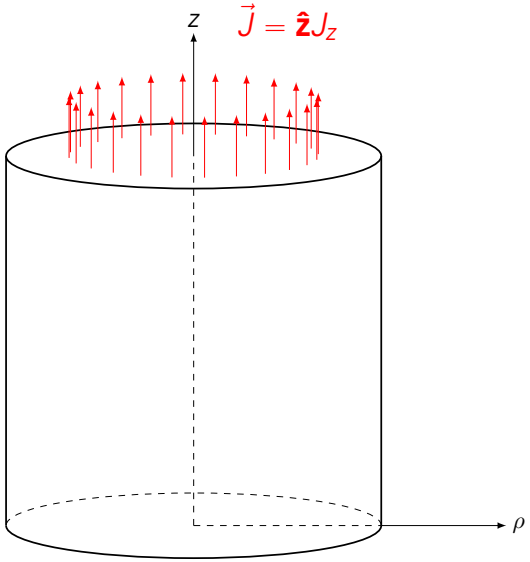
2. Ampere's Law: $\oint_C \vec{H} \cdot \vec{d\ell} = I_{encls}$

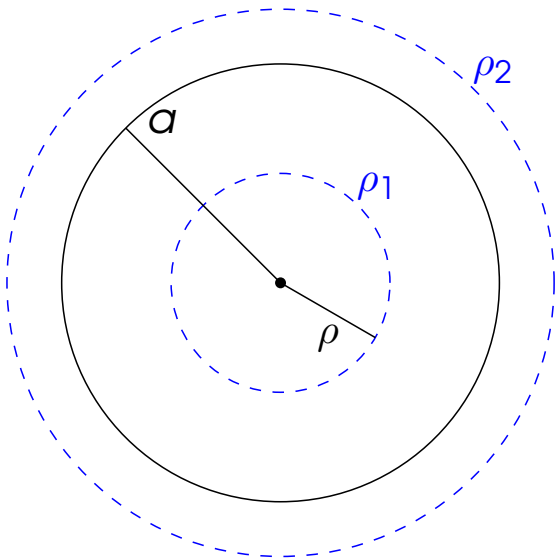
3. $\vec{H} = \hat{\boldsymbol{\rho}} H_\rho + \hat{\boldsymbol{\phi}} H_\phi + \hat{\mathbf{z}} H_z$

4. $\vec{d\ell} = \hat{\boldsymbol{\phi}} \rho d\phi = \hat{\boldsymbol{\phi}} b d\phi$

5. $\oint_C \vec{H} \cdot \vec{d\ell} = \oint_0^{2\pi} H_\phi b d\phi$

Ampere's Law: Coaxial Cable





1. Draw picture(s)
2. Ampere's Law: $\oint_C \vec{H} \cdot \vec{d\ell} = I_{\text{encls}}$
3. $\vec{H} = \hat{\boldsymbol{\rho}} H_\rho + \hat{\boldsymbol{\phi}} H_\phi + \hat{\mathbf{z}} H_z$
4. $\vec{d\ell} = \hat{\boldsymbol{\phi}} \rho d\phi$
5. $\oint_C \vec{H} \cdot \vec{d\ell} = \int_0^{2\pi} H_\phi \rho d\phi$

Ampere's Law: Coaxial Cable

$$\textcircled{6}\ I_{encls} = \iint_S \vec{J}\cdot d\vec{S}$$

$$= \int_0^{2\pi}\int_0^{\rho_0} \hat{\mathbf{z}} J_z \hat{\mathbf{z}} \rho d\rho d\phi$$

$$= (2\pi J_z)\left(\frac{\rho_0^2}{2}\right)$$

$$H_\phi = J_z \frac{\pi\rho_0^2}{2\pi\rho}$$

Case 1: $\rho < a$

$$\rho_0 = \rho$$

$$H_\phi = J_z \frac{\rho}{2}$$

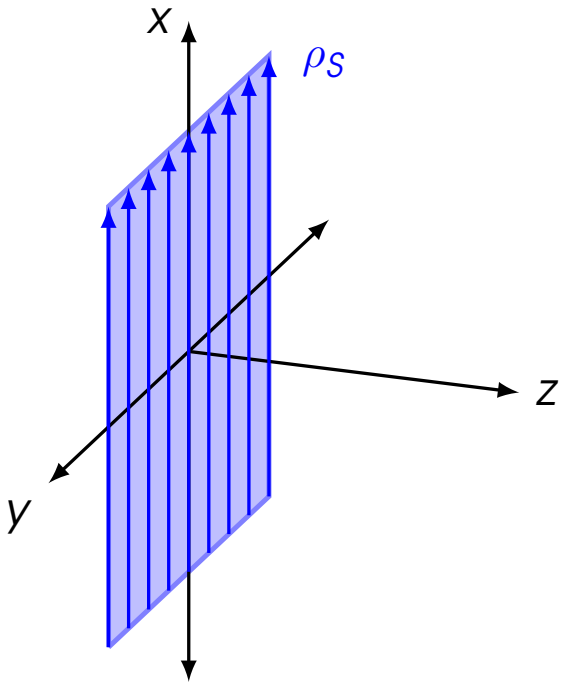
Case 2: $\rho > a$

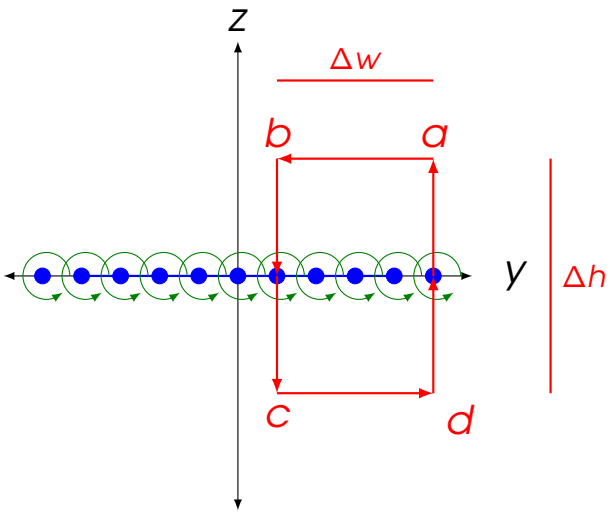
$$\rho_0 = a$$

$$H_\phi = J_z \frac{a^2}{2\rho}$$

Now, plot H_ϕ...

Ampere's Law: Sheet Charge





1. Draw picture(s)

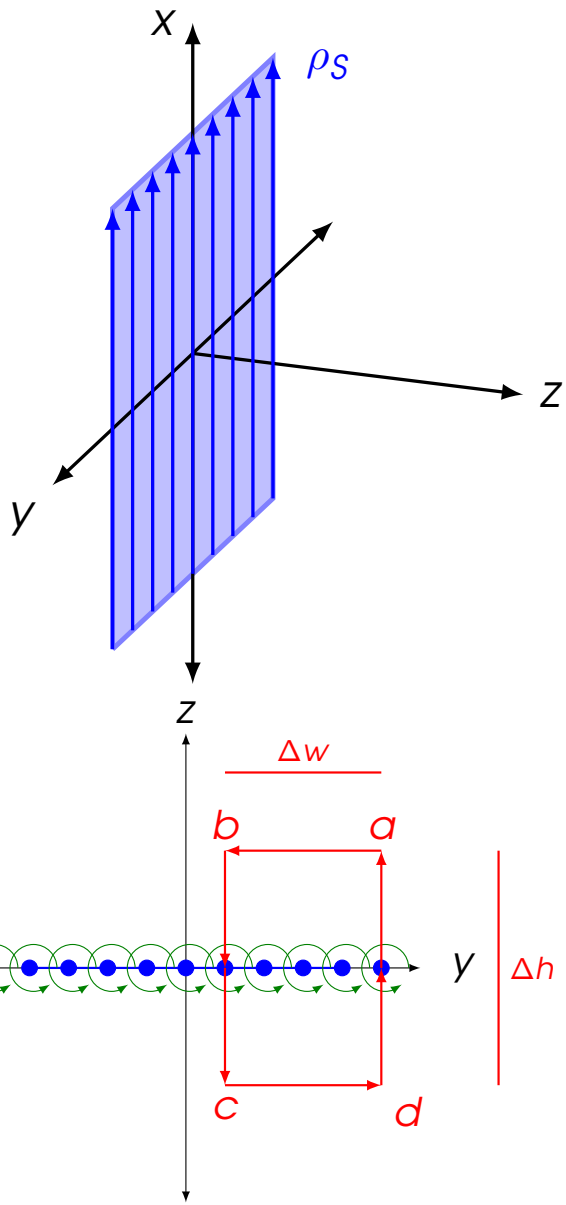
2. Ampere's Law: $\oint_C \vec{H} \cdot d\vec{\ell} = I_{\text{encls}}$

3. $\vec{H} = \hat{\mathbf{x}} H_x + \hat{\mathbf{y}} H_y + \hat{\mathbf{z}} H_z$

4. $\oint_C \vec{H} \cdot d\vec{\ell} = \int_a^b \vec{H} \cdot d\vec{\ell}_1 + \int_b^c \vec{H} \cdot d\vec{\ell}_2$

$$\int_c^d \vec{H} \cdot d\vec{\ell}_3 + \int_d^a \vec{H} \cdot d\vec{\ell}_4$$

Ampere's Law: Sheet Charge



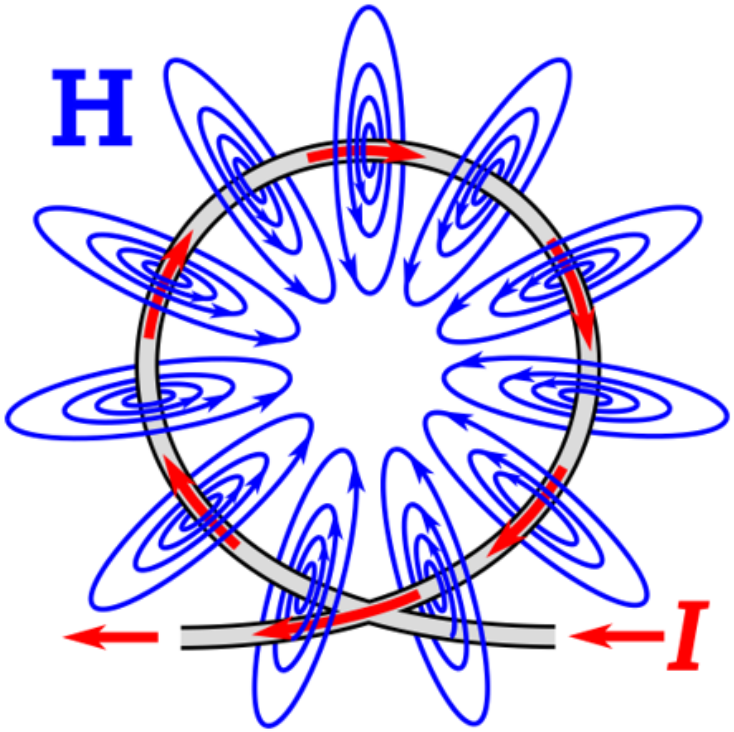
$$\textcircled{6}\ I_{encls} = \iint_S \vec{J}\cdot\vec{dS}$$

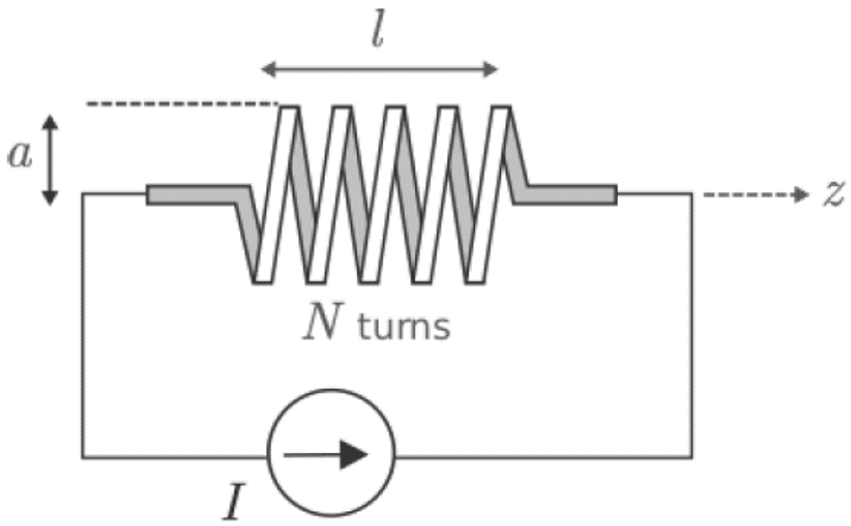
$$= \int_0^{\Delta w} \hat{\mathbf{x}} J_x \cdot \hat{\mathbf{x}}\, dy$$

$$= J_x \Delta w$$

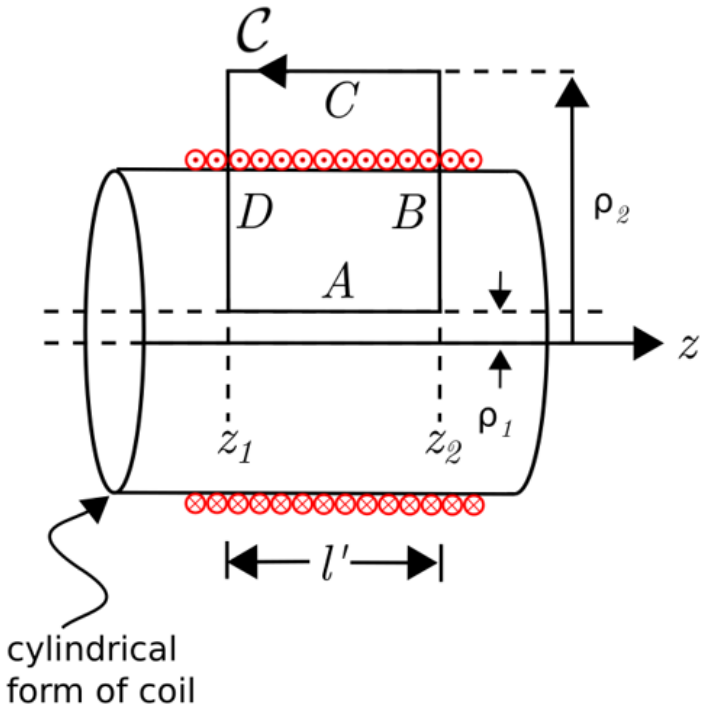
$$H_y = \frac{J_x}{2}$$

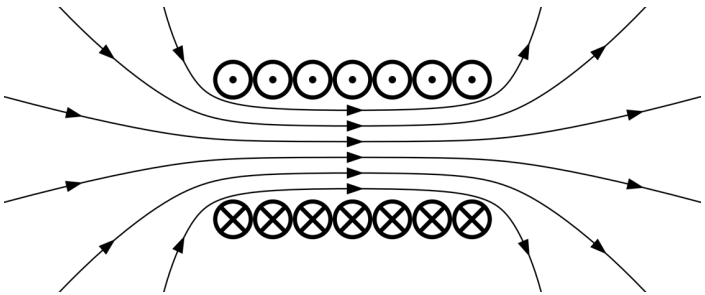
Ampere's Law: Solenoid





Ampere's Law: Solenoid

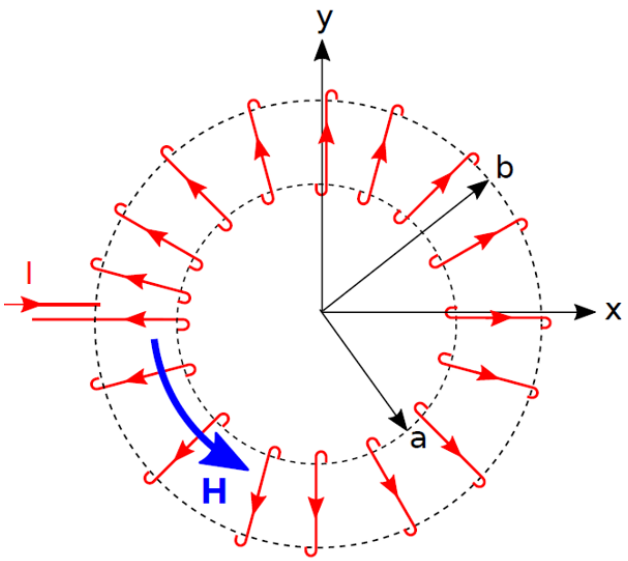




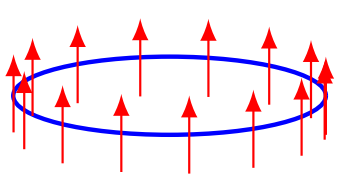
1. Draw picture(s)
2. Ampere's Law: $\oint_C \vec{H}\cdot d\vec{\ell} = I_{\text{encls}}$
3. $\vec{H} = \hat{\boldsymbol{\rho}} H_\rho + \hat{\boldsymbol{\phi}} H_\phi + \hat{\mathbf{z}} H_z$
4. $\oint_C \vec{H}\cdot d\vec{\ell} = \int_A \vec{H}\cdot d\vec{\ell}_1 + \int_B \vec{H}\cdot d\vec{\ell}_2$

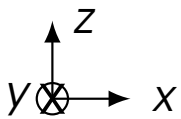
$$\int_C \vec{H}\cdot d\vec{\ell}_3 + \int_D \vec{H}\cdot d\vec{\ell}_4$$

Ampere's Law: Toroid



Note: the contour of integration is **inside** the core, meaning current through the surface defined by the contour is carried in the windings at the inner edge of the core:





1. Draw picture(s)

2. Ampere's Law: $\oint_C \vec{H}\cdot d\vec{\ell} = I_{\text{encls}}$

3. $\vec{H} = \hat{\boldsymbol{\rho}} H_\rho + \hat{\boldsymbol{\phi}} H_\phi + \hat{\mathbf{z}} H_z$

4. $\oint_C \vec{H}\cdot d\vec{\ell} = \int_0^{2\pi} \vec{H}\cdot d\vec{\ell}_\phi$

$$= \int_0^{2\pi} \left(\hat{\boldsymbol{\phi}} H_\phi\right)\cdot\left(\hat{\boldsymbol{\phi}}\rho d\phi\right)$$

$$= 2\pi\rho H_\phi$$

5. $I_{\text{encls}} = NI$

$$\vec{H} = \hat{\boldsymbol{\phi}} \frac{NI}{2\pi\rho}$$

Maxwell's Equations: Static Fields

Law	Integral Form	Point Form
Gauss's Law for Electricity	$\oint_S \vec{E} \cdot \vec{dS} = \frac{Q_{\text{encls}}}{\varepsilon}$	$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_V}$
Gauss's Law for Magnetism	$\oint_S \vec{B} \cdot \vec{dS} = 0$	$\nabla \cdot \vec{B} = 0$
Faraday's Law	$\oint_S \vec{E} \cdot \vec{d\ell} = 0$	$\nabla \times \vec{E} = 0$
Ampere's Law	$\oint_S \vec{H} \cdot \vec{d\ell} = I_{\text{encl}}$	$\nabla \times \vec{H} = \vec{J}$