ECE343 Spring 2025

Objective 6.1: Current Density



UNITED STATES AIR FORCE ACADEMY

Lesson	Topics	Reading	Notes
24	BVP (Capacitor, Coax), Electric Potential	5.16	LO 5.3
25	Electric Boundary Conditions	5.18-5.22	LO 5.4
26	BC's/Capacitance	5.23-5.24	
27	Electrostatic Lab	Lab packet	
Chapter 6: Steady Current & Conductivity			
28	Current density, Conductivity, Resistance	Ch. 6	
Chapter 7: Magnetostatics			
29	Magnetic Fields, Gauss' Law for Magnetism	7.1-7.3	LO 7.1-7.2 **GR 2**
30	Ampere's Law	7.4-7.5, 7.9	LO 7.3
31	Ampere's Law Applied	7.6-7.8	LO 7.3
32	Magnetic Boundary Conditions	7.10-7.14	LO 7.4
33	Magnetostatics Lab	Lab packet	
Chapter 8: Time-Varying Fields			
34	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.1
35	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.2-8.3
Chapter 9: Plane Waves in Lossless Media			
36	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
37	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
38	Uniform Plane Waves & Polarization	9.4-9.6	LO 9.2-9.3
39	Normal Incidence	Vol 2, 5.6-5.7	LO 9.4-9.5
40	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
41	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
42	Incidence Lab	TBD	LO 9.5

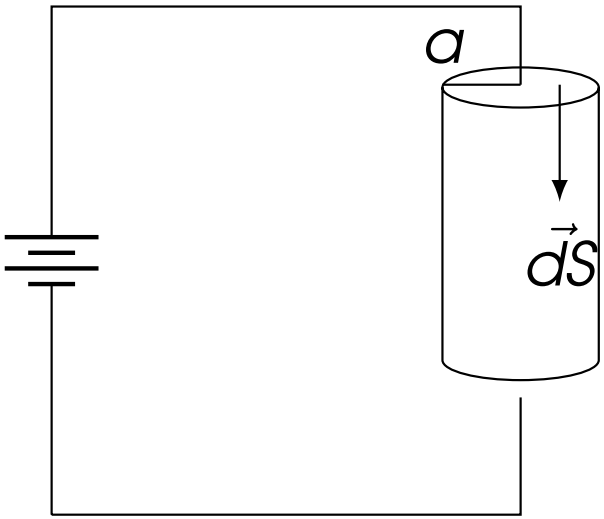
Ch 6 Learning Objectives

5.1 I can calculate the total current through a surface area given a current distribution.

5.2 I can use Ohm's Law for electromagnetics to calculate the relationship between a current distribution and an electric field.

5.3 I can articulate and identify what makes a material a good conductor and how conductivity relates to resistance.

Current Distributions



$$I = \int_0^{\rho=a} \int_{\phi=0}^{2\pi} \vec{J} \cdot d\vec{S}$$

$$= 8\text{A/m}^2 \int_0^{\rho=a} \int_{\phi=0}^{2\pi} \rho d\rho d\phi$$

$$= \left(8\text{A/m}^2\right)\left(\frac{a^2}{2}\right)(2\pi)$$

$$= 226\mu\text{A}$$

Conductivity

Definition

$$\vec{J} = \sigma \vec{E}$$

$$\Longrightarrow \quad \sigma \longrightarrow \frac{1}{\Omega} = \text{S (Siemens)}$$

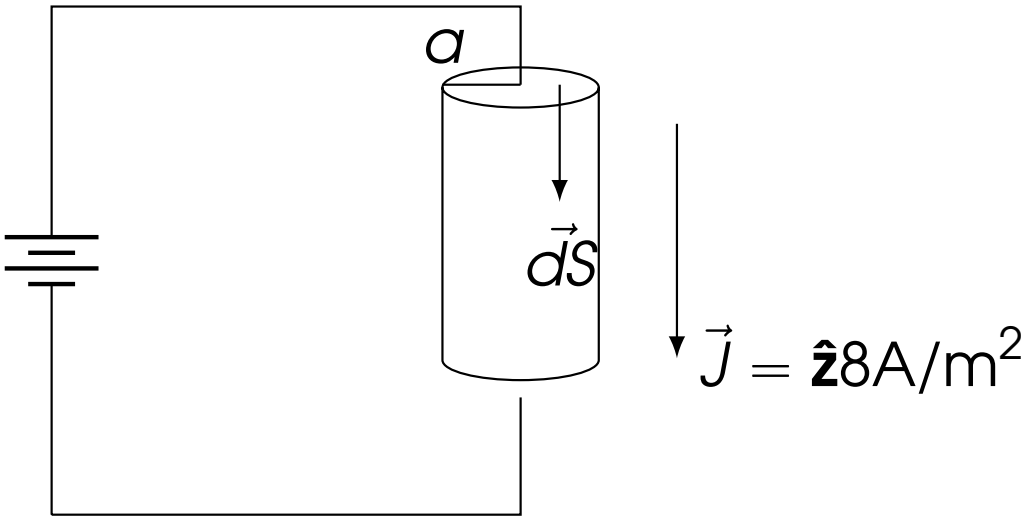
Types of Conductors

1. "Good" conductor $\longrightarrow \sigma > 10^5$ S/m
2. Semiconductor $\longrightarrow \times 10^{-4} < \sigma < 10$ S/m
3. Poor insulators $\longrightarrow 10$ S/m
4. Good insulators $\longrightarrow \sigma < \times 10^{-10}$ S/m
5. Vacuum $\longrightarrow 0$

Conductivity for Common Materials

Material	σ (S/m)
Copper	5.8×10^7
Gold	4.4×10^7
Aluminum	3.7×10^7
Iron	1.0×10^7
Platinum	0.9×10^7
Carbon	1.3×10^5
Silicon	4.4×10^{-4}
Free space	≈ 0
Distilled water	6×10^{-6}
Tap water	$5 - 50 \times 10^{-3}$
Seawater	5
Soil	1×10^{-4}

Resistance of a Wire



$$V = -\int \vec{E} \cdot d\vec{\ell}$$

$$= -\int (-\hat{\mathbf{z}} E_z) \cdot \hat{\mathbf{z}} dz$$

$$= E_z \ell$$

$$\Longrightarrow \vec{J} = \sigma \vec{E} = \hat{\mathbf{z}} \sigma E_z$$

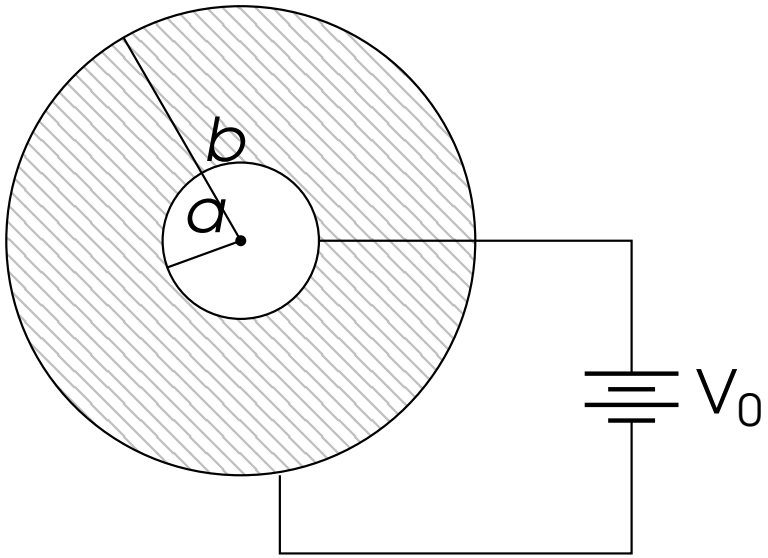
$$\Longrightarrow I = \int_{\rho=0}^{a} \int_{\phi=0}^{2\pi} \hat{\mathbf{z}} \sigma E_z \cdot \hat{\mathbf{z}} \rho d\rho d\phi$$

$$= \sigma E_z \pi a^2$$

$$\Longrightarrow R = \frac{V}{I} = \frac{E_z \ell}{\sigma E_z \pi a^2} = \frac{\ell}{\sigma A}$$

Resistance of a Coaxial Cable

Find the resistance between the inner and outer conductor of a coaxial cable of length (in z-dimension) ℓ.



$$\vec{E} = \hat{\rho}\frac{Q}{2\pi\varepsilon_0\rho L}$$

$$V = -\int_b^a \frac{Q}{2\pi\varepsilon_0\rho L}$$

$$= \frac{Q}{2\pi\varepsilon_0\rho L}\ln\left(\frac{b}{a}\right)$$

$$I = \int \vec{J}\cdot\vec{dS} = \int \sigma\vec{E}\cdot\vec{dS}$$

$$= \int \sigma\hat{\boldsymbol{\rho}}\frac{Q}{2\pi\varepsilon_0\rho L}\cdot\hat{\boldsymbol{\rho}}\rho d\phi dz$$

$$= \frac{\sigma Q}{2\pi\varepsilon_0 L}\int_0^{2\pi} d\phi\int_0^L dz = \frac{\sigma Q}{\varepsilon_0}$$

$$R = \frac{V}{I} = \left[\frac{Q}{2\pi\varepsilon_0\rho L}\ln\left(\frac{b}{a}\right)\right]\left[\frac{\varepsilon_0}{\sigma Q}\right]$$

$$= \frac{1}{2\pi\sigma L}\ln\left(\frac{b}{a}\right)$$

$$\Longrightarrow G' = \frac{G}{L} = \frac{1}{RL} = \frac{2\pi\sigma}{\ln\left(\frac{b}{a}\right)}$$