ECE343 Spring 2025

Objective 5.4: Electric field Boundary Conditions



Lesson	Topics	Reading	Notes
24	BVP (Capacitor, Coax), Electric Potential	5.16	LO 5.3
25	Electric Boundary Conditions	5.18-5.22	LO 5.4
26	BC's/Capacitance	5.23-5.24	
27	Electrostatic Lab	Lab packet	
Chapter 6: Steady Current & Conductivity			
28	Current density, Conductivity, Resistance	Ch. 6	
Chapter 7: Magnetostatics			
29	Magnetic Fields, Gauss' Law for Magnetism	7.1-7.3	LO 7.1-7.2 **GR 2**
30	Ampere's Law	7.4-7.5, 7.9	LO 7.3
31	Ampere's Law Applied	7.6-7.8	LO 7.3
32	Magnetic Boundary Conditions	7.10-7.14	LO 7.4
33	Magnetostatics Lab	Lab packet	
Chapter 8: Time-Varying Fields			
34	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.1
35	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.2-8.3
Chapter 9: Plane Waves in Lossless Media			
36	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
37	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
38	Uniform Plane Waves & Polarization	9.4-9.6	LO 9.2-9.3
39	Normal Incidence	Vol 2, 5.6-5.7	LO 9.4-9.5
40	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
41	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
42	Incidence Lab	TBD	LO 9.5

Ch 5 Learning Objectives

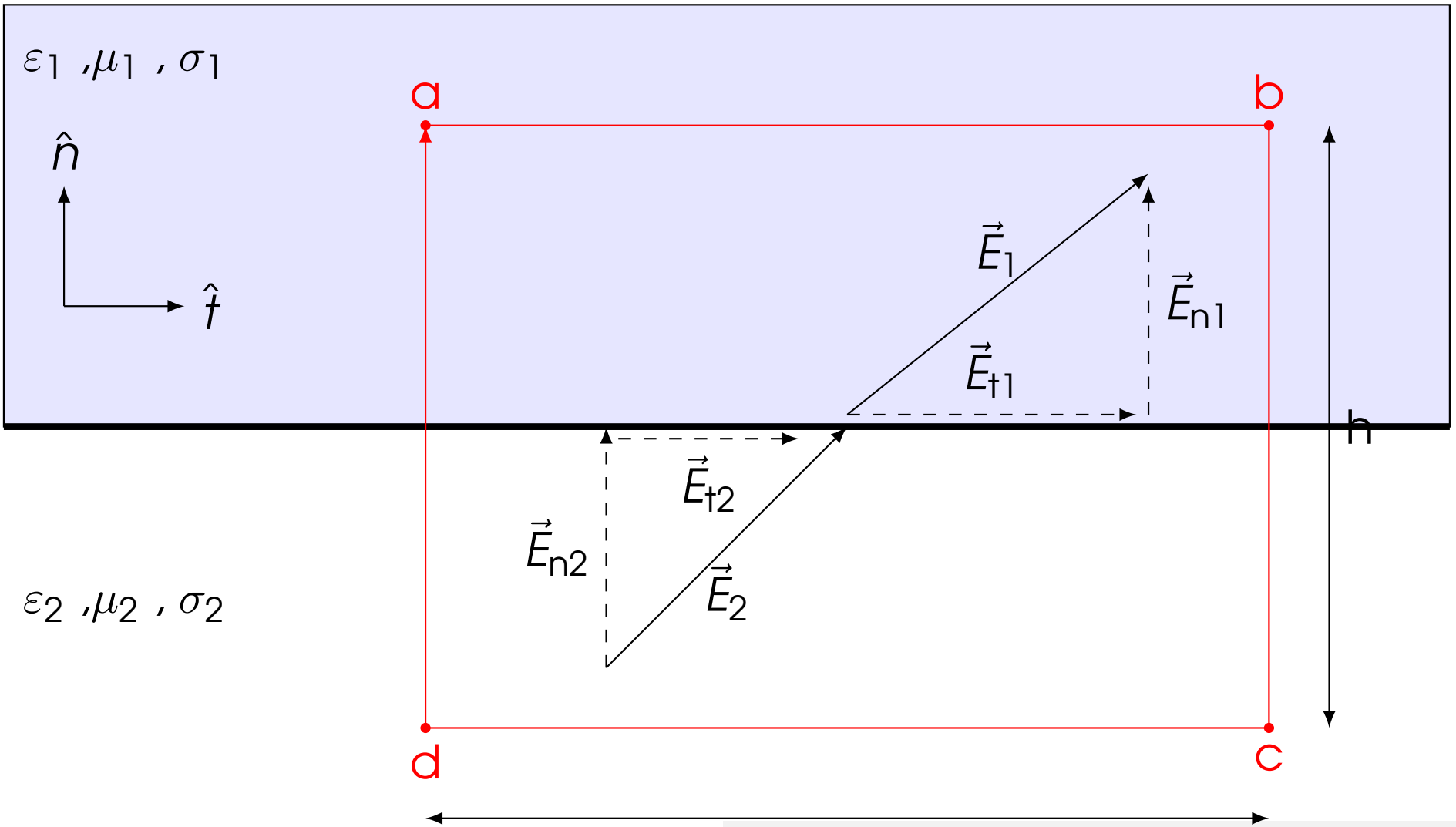
5.1 I can apply Coloumb's Law to use direct integration to calculate the electric field due to charge density in varoius geometries, including rectangular, cylindrical, and spherical coordinate systems.

5.2 I can apply the integral and differential forms of Gauss' Law to calculate the electric field due to charge density in various geometries, including rectangular, cylindrical, and spherical coordinate systems.

5.3 I can calculate the electric field from the scalar electric potential in a variety of geometries.

5.4 I can apply the electric field boundary conditions to calculate electric fields on both sides of a material boundary.

Static Electric Fields with Boundary



Boundary Conditions: Transverse

$$\oint \vec{\boldsymbol{E}} \cdot d\vec{\ell} = 0$$

$$\int \vec{\boldsymbol{E}} \cdot d\vec{\ell}_{ab} + \int \vec{\boldsymbol{E}} \cdot d\vec{\ell}_{bc} + \int \vec{\boldsymbol{E}} \cdot d\vec{\ell}_{cd} + \int \vec{\boldsymbol{E}} \cdot d\vec{\ell}_{da} = 0$$

$$\int \vec{\boldsymbol{E}} \cdot d\vec{\ell}_{ab} = \int_a^b \vec{\boldsymbol{E}}_{t1} \cdot \hat{t} d\ell = E_{t1} W$$

$$\int \vec{\boldsymbol{E}} \cdot d\vec{\ell}_{bc} = \int_{b-h/2}^{b} \vec{\boldsymbol{E}}_1 \cdot \left(-\hat{n}\right) d\ell + \int_{b-h/2}^{c} \vec{\boldsymbol{E}}_2 \cdot \left(-\hat{n}\right) d\ell = -\left(E_{n1} + E_{n2}\right) \frac{h}{2}$$

$$\int \vec{\boldsymbol{E}} \cdot d\vec{\ell}_{cd} = \int_c^d \vec{\boldsymbol{E}}_{t2} \cdot \left(-\hat{t}\right) d\ell = -E_{t2} W$$

$$\int \vec{\boldsymbol{E}} \cdot d\vec{\ell}_{da} = \int_d^{d+h/2} \vec{\boldsymbol{E}}_2 \cdot \left(\hat{n}\right) d\ell + \int_{d+h/2}^{a} \vec{\boldsymbol{E}}_1 \cdot \left(\hat{n}\right) d\ell = \left(E_{n1} + E_{n2}\right) \frac{h}{2}$$

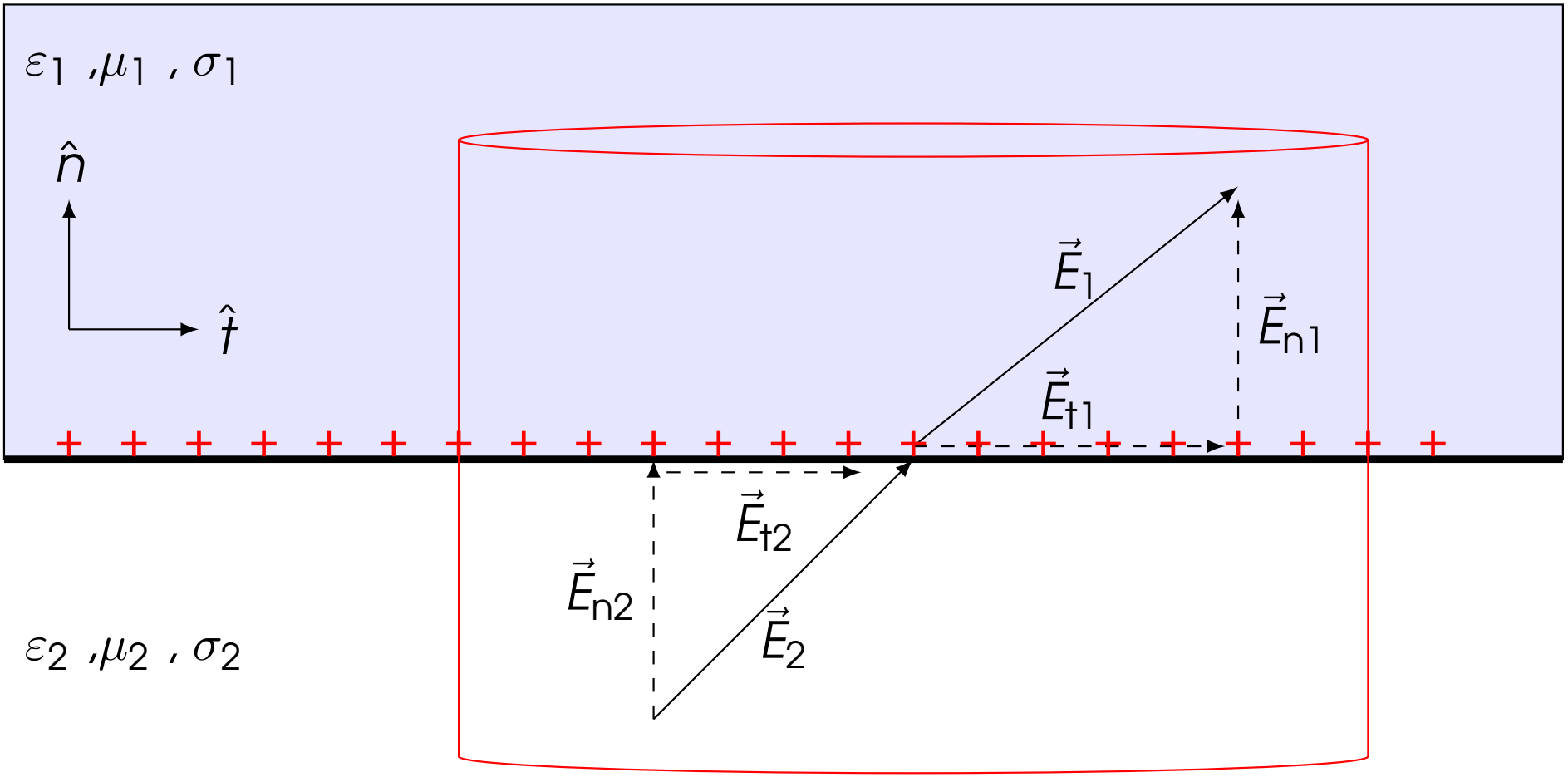
$$\implies \oint \vec{\boldsymbol{E}} \cdot d\vec{\ell} = E_{t1} W + -\left(E_{n1} + E_{n2}\right) \frac{h}{2} - E_{t2} W + \left(E_{n1} + E_{n2}\right) \frac{h}{2} = 0$$

Boundary Conditions: Transverse

Transverse

$$E_{t1} = E_{t2} \qquad \text{or} \qquad \hat{n}_{21} \times \left(\vec{E}_1 - \vec{E}_2 \right) = 0$$

Boundary Conditions: Normal



$$\Longrightarrow \oiint \vec{D} \cdot d\vec{S} = \rho_S$$

Boundary Conditions: Normal

$$\text{Top} \longrightarrow \oiint \vec{D}\cdot d\vec{S}_{\text{top}} = \oiint \vec{D}\cdot \hat{n} dS = D_{\text{n1}}(\text{area})$$

$$\text{Bottom} \longrightarrow \oiint \vec{D}\cdot d\vec{S}_{\text{bottom}} = \oiint \vec{D}\cdot \left(-\hat{n}\right) dS = -D_{\text{n2}}(\text{area})$$

$$\text{Sides} \xrightarrow{h\to 0} \oiint \vec{D}\cdot d\vec{S}_{\text{sides}} = 0 \qquad Q_{\text{enclosed}}$$

$$\Longrightarrow D_{\text{n1}} - D_{\text{n2}} = \rho_S \qquad \text{or} \qquad \hat{n}_{21}\cdot\left(\varepsilon_1\vec{E}_1 - \varepsilon_2\vec{E}_2\right) = \rho_S$$

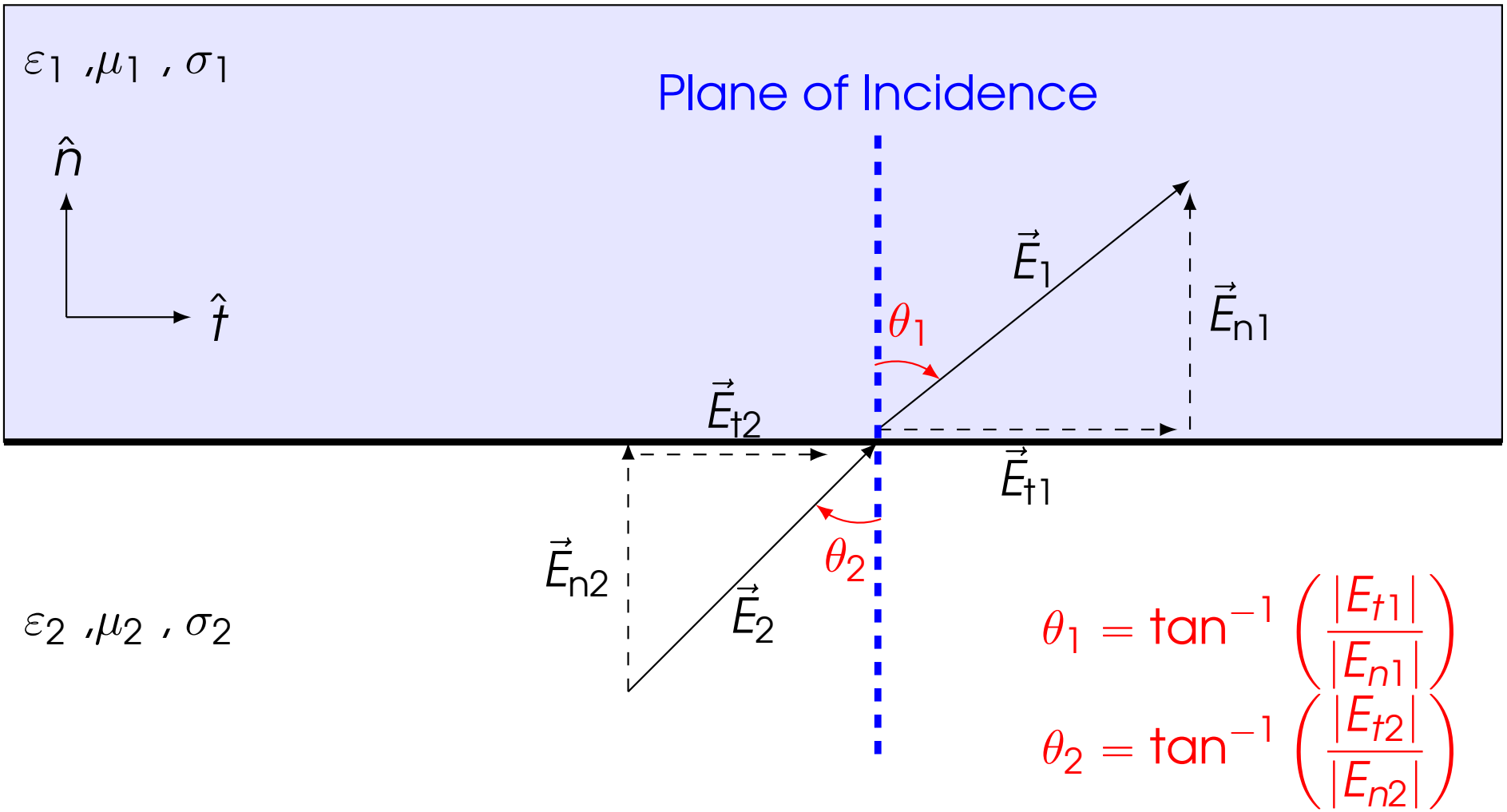
Boundary Conditions: Summary

Electric Field Boundary Conditions

Transverse: $$\hat{n}_{21} \times \left(\vec{E}_1 - \vec{E}_2 \right) = 0$$

Normal: $$\hat{n}_{21} \cdot \left(\varepsilon_1 \vec{E}_1 - \varepsilon_2 \vec{E}_2 \right) = \rho_S$$

Angle of Incidence

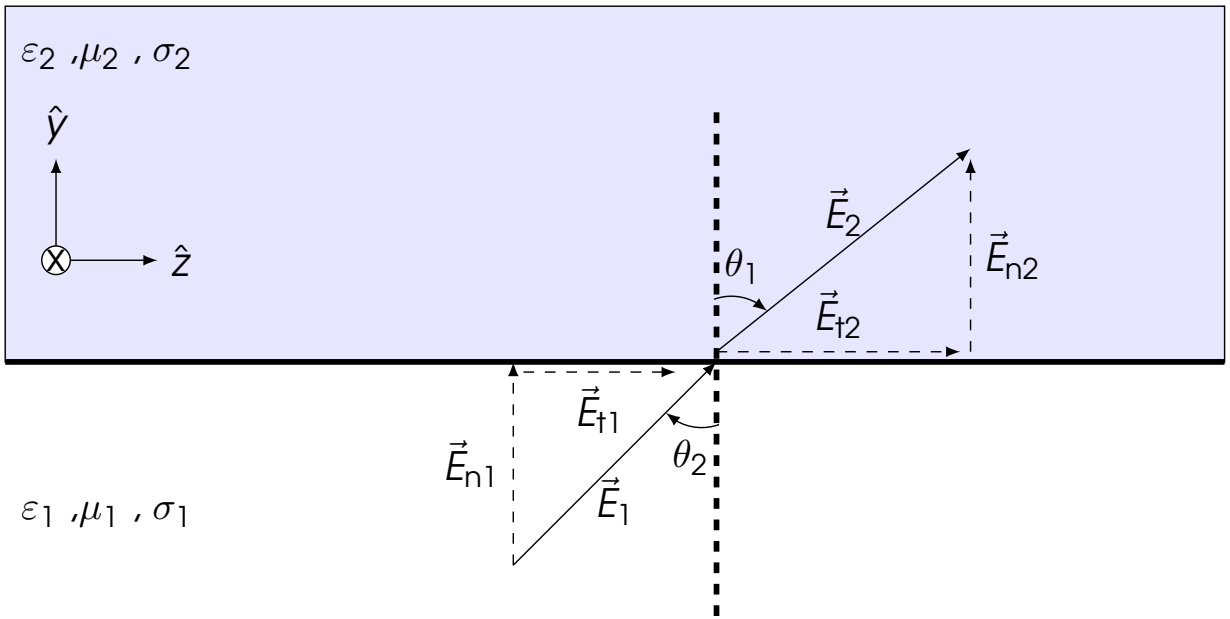


$$\theta_1 = \tan^{-1}\left(\frac{|E_{t1}|}{|E_{n1}|}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{|E_{t2}|}{|E_{n2}|}\right)$$

Boundary Conditions: Example

An electric field $\vec{E}_1$ is incident from free space onto a material with $\varepsilon_{2r} = 2$. If $\vec{E}_2 = \hat{\mathbf{x}}3 + \hat{\mathbf{y}}4 + \hat{\mathbf{z}}5$, find $\vec{E}_1$ and the angle of incidence of each vector.



Transverse $\longrightarrow \hat{\mathbf{x}}, \hat{\mathbf{z}}$

Normal $\longrightarrow \hat{\mathbf{y}}$

Boundary Conditions: Example (cont'd)

(1) $\begin{cases} \vec{E}_{t2} = \hat{\mathbf{x}}3 + \hat{\mathbf{z}}5 \implies |\vec{E}_{t2}| = \sqrt{3^2 + 5^2} = \sqrt{34} \\ \vec{E}_{n2} = \hat{\mathbf{y}}4 \implies |\vec{E}_{n2}| = 4 \end{cases}$

(2) $\longrightarrow \theta_2 = \tan^{-1}\left(\dfrac{|\vec{E}_{t2}|}{|\vec{E}_{n2}|}\right) = \tan^{-1}\left(\dfrac{\sqrt{34}}{4}\right) = 55.6^\circ$

(3) $\longrightarrow$ Normal BC $\implies \hat{n}_{2,1} = -\hat{\mathbf{y}}$

$$-\hat{\mathbf{y}}\cdot\left(\varepsilon_1\vec{E}_1 - \varepsilon_2\vec{E}_2\right) = 0$$

$$-\hat{\mathbf{y}}\cdot\varepsilon_0\vec{E}_1 + \hat{\mathbf{y}}\cdot\left[2\varepsilon_0\left(\hat{\mathbf{x}}3 + \hat{\mathbf{y}}4 + \hat{\mathbf{z}}5\right)\right] = 0$$

$$-\varepsilon_0 E_{1y} + 8\varepsilon_0 = 0$$

$$\implies E_{1y} = 8$$

Boundary Conditions: Example (cont'd)

(4) $\longrightarrow$ Transverse BC

(5) $\longrightarrow \vec{E}_1 = \hat{\mathbf{x}}3 + \hat{\mathbf{y}}8 + \hat{\mathbf{z}}5$

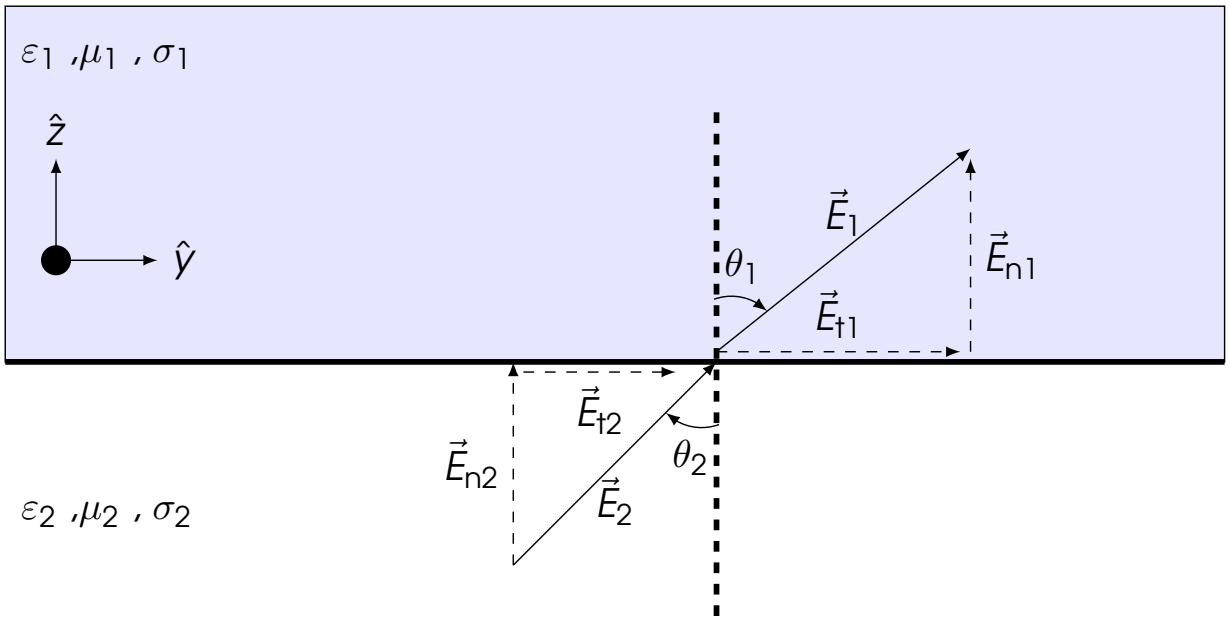
(6) $\longrightarrow \theta_1 = \tan^{-1}\left(\frac{|\vec{E}_{t1}|}{|\vec{E}_{n1}|}\right) = \tan^{-1}\left(\frac{\sqrt{34}}{8}\right) = 36^\circ$

Summary

$$\vec{E}_1 = \hat{\mathbf{x}}3 + \hat{\mathbf{y}}8 + \hat{\mathbf{z}}5$$
$$\vec{E}_2 = \hat{\mathbf{x}}3 + \hat{\mathbf{y}}4 + \hat{\mathbf{z}}5$$
$$\theta_1 = 36^\circ$$
$$\theta_2 = 55.6^\circ$$

YOUR TURN: EXAMPLE #2

An electric field $\vec{E}_1$ is incident from free space onto a material with $\varepsilon_{2r} = 4$. If $\vec{E}_2 = \hat{\mathbf{x}}3 + \hat{\mathbf{y}}4 + \hat{\mathbf{z}}5$, find $\vec{E}_1$ and the angle of incidence of each vector.

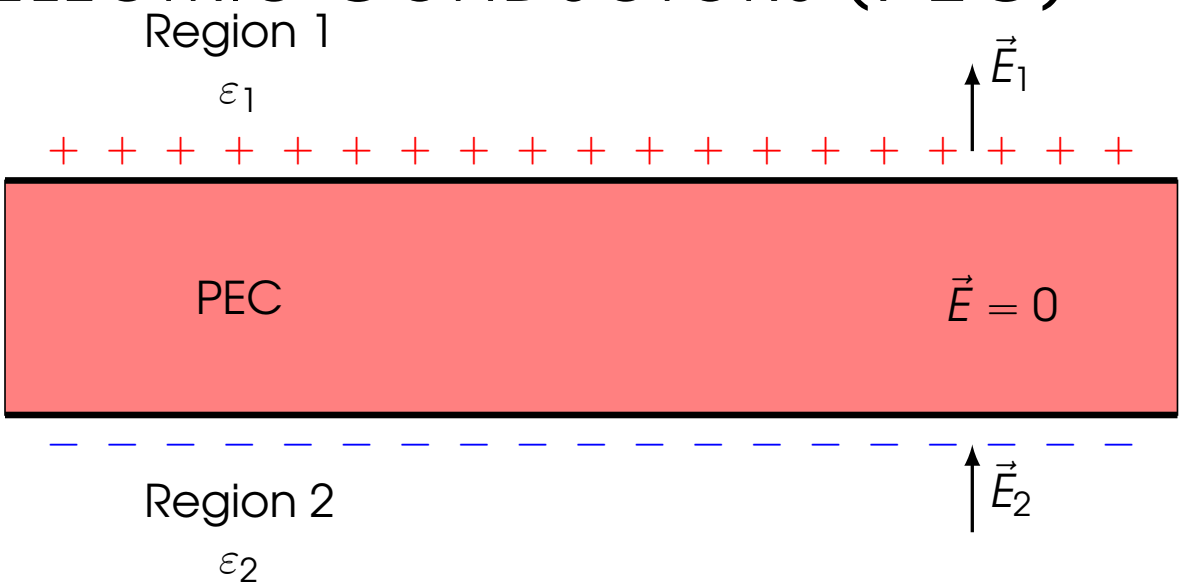


Transverse $\longrightarrow$

Normal $\longrightarrow$

Your Turn: Example #2

Perfect Electric Conductors (PEC)



Electric Field in a PEC

1. $\vec{E} = 0$ inside a PEC
2. If $\vec{E}_{\text{PEC}} = 0$, then $\vec{E}_{\text{t,outside}} = \vec{E}_{\text{t,PEC}} = 0$
3. Normal BC requires $\vec{E}_{\text{n, outside}} = \dfrac{\rho_S}{\varepsilon}$
4. No static charge **inside** PEC

Capacitance

Capacitance

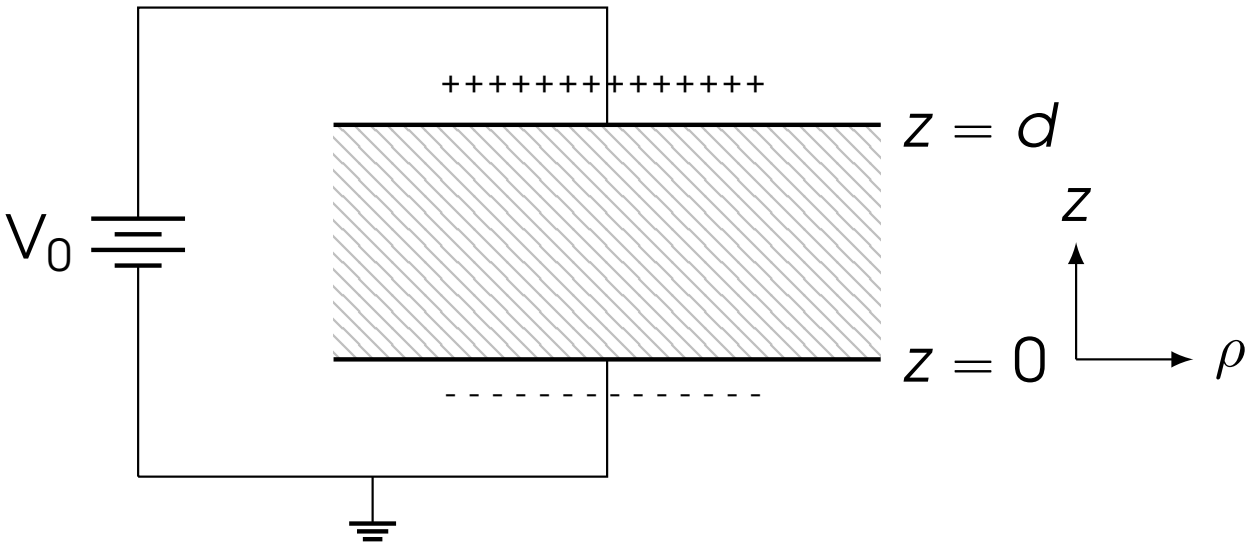
1. Capacitance is the ability of a structure to store energy in an electric field $\longrightarrow C = \dfrac{Q}{V}$
2. Depends on the geometry and permittivity of medium separating plates.

$$Q(t) = \int_{t_0}^{t} I(\tau)\,d\tau + Q(t_0)$$

$$\Longrightarrow V(t) = \frac{1}{C}\int_{t_0}^{t} I(\tau)\,d\tau + \frac{1}{C}Q(t_0) = \frac{1}{C}\int_{t_0}^{t} I(\tau)\,d\tau + V_0$$

$$\Longrightarrow I(t) = C\frac{\partial}{\partial t}V(t)$$

Parallel Plate Capacitor



$$V = -\int_C \vec{E} \cdot d\ell$$

$$= -\int_0^d \frac{1}{\varepsilon} \vec{D} \cdot (\hat{\mathbf{z}} dz)$$

$$= -\int_0^d \left(-\hat{\mathbf{z}} \frac{\rho_s}{\varepsilon}\right) \cdot (\hat{\mathbf{z}} dz)$$

$$V = -\frac{\rho_s d}{\varepsilon}$$

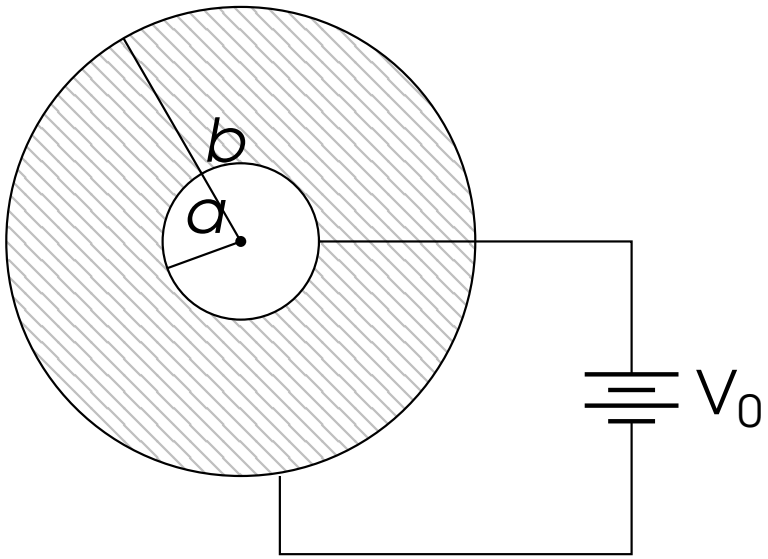
Now the capacitance is:

$$C = \frac{Q}{V} = \frac{\rho_s A}{\frac{\rho_s d}{\varepsilon}}$$

$$= \frac{\varepsilon A}{d}$$

Coaxial Capacitor

Find the capacitance for a coaxial capacitor of length (in z-dimension) ℓ.



$$\vec{E} = \hat{\boldsymbol{\rho}}\frac{\rho_\ell}{2\pi\varepsilon\rho}$$

$$Q = \rho_\ell \ell$$

$$\begin{aligned} V &= -\int_C \vec{E}\cdot d\vec{\ell} \\ &= -\int_{\rho=a}^{b}\left(\hat{\boldsymbol{\rho}}\frac{\rho_\ell}{2\pi\varepsilon\rho}\right)\cdot(\hat{\boldsymbol{\rho}}d\rho) \\ &= -\frac{\rho_\ell}{2\pi\varepsilon}\int_{\rho=a}^{b}\frac{1}{\rho}d\rho \\ &= \frac{\rho_\ell}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right) \end{aligned}$$

$$\Longrightarrow C = \frac{Q}{V} = \frac{\rho_\ell \ell}{\frac{\rho_\ell}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)}$$

$$= \frac{2\pi\varepsilon\ell}{\ln\left(\frac{b}{a}\right)}$$