ECE343 Spring 2025

Objective 5.1: Coloumb's Law



Lesson	Topics	Reading	Notes
1	Course Overview & Introduction	Ch.1	LO 1.1-1.2
Chapter 2: Electric and Magnetic Fields			
2	Fields, Parameters, Field Quantities	Ch. 2	LO 2.1-2.2
Chapter 3: Transmission Lines			
3	Lumped Element Model	3.1-3.4	LO 3.1
4	Telegrapher's Equations	3.5-3.6	LO 3.1
5	Characteristic Impedance & Wave Propagation	3.7-3.9	LO 3.2
6	Coaxial, Reflection Coefficient, VSWR	3.10,3.12-3.14	LO 3.2-3.3
7	Terminated Transmission Lines	3.15,3.17-3.19	LO 3.4
8	Impedance Matching	3.21-3.23	LO 3.4
9	Impedance Matching	3.21-3.23	LO 3.4
10	Smith Charts	Supplemental	LO 3.5
11	Smith Charts	Supplemental	LO 3.5
12	Smith Charts	Supplemental	LO 3.5
13	Smith Charts	Supplemental	LO 3.5
14	Transients	Supplemental	LO 3.6
15	Flex Day		
16	Reflection/Transmission Coefficient Lab	Lab Packet	LO 3.7
17	Reflection/Transmission Coefficient Lab	Lab Packet	LO 3.7
Chapter 4: Vector Analysis			
18	Vector & Coordinate System Review	Ch. 4	LO 4.1-4.4 **GR 1 due**
Chapter 5: Electrostatics			
19	Coulomb's Law	5.1-5.3	LO 5.1
20	Surface and Volume Charge	5.4	LO 5.1
21	Gauss' Law (Integral Form)	5.5-5.6	LO 5.2
22	Gauss' Law (Differential Form)	5.7-5.8	LO 5.2
23	Electric Potential	5.9-5.10, 5.15	LO 5.3

Lesson	Topics	Reading	Notes
24	BVP (Capacitor, Coax), Electric Potential	5.16	LO 5.3
25	Electric Boundary Conditions	5.18-5.22	LO 5.4
26	BC's/Capacitance	5.23-5.24	
27	Electrostatic Lab	Lab packet	
Chapter 6: Steady Current & Conductivity			
28	Current density, Conductivity, Resistance	Ch. 6	
Chapter 7: Magnetostatics			
29	Magnetic Fields, Gauss' Law for Magnetism	7.1-7.3	LO 7.1-7.2 **GR 2**
30	Ampere's Law	7.4-7.5, 7.9	LO 7.3
31	Ampere's Law Applied	7.6-7.8	LO 7.3
32	Magnetic Boundary Conditions	7.10-7.14	LO 7.4
33	Magnetostatics Lab	Lab packet	
Chapter 8: Time-Varying Fields			
34	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.1
35	Faraday's Law, Displacement Current, Maxwell Eqns	8.3, 8.8, 8.9	LO 8.2-8.3
Chapter 9: Plane Waves in Lossless Media			
36	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
37	Maxwell's Equations in lossless, source-free region	9.1-9.2	LO 9.1
38	Uniform Plane Waves & Polarization	9.4-9.6	LO 9.2-9.3
39	Normal Incidence	Vol 2, 5.6-5.7	LO 9.4-9.5
40	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
41	Oblique Incidence	Vol 2, 5.6-5.7	LO 9.5
42	Incidence Lab	TBD	LO 9.5

Ch 5 Learning Objectives

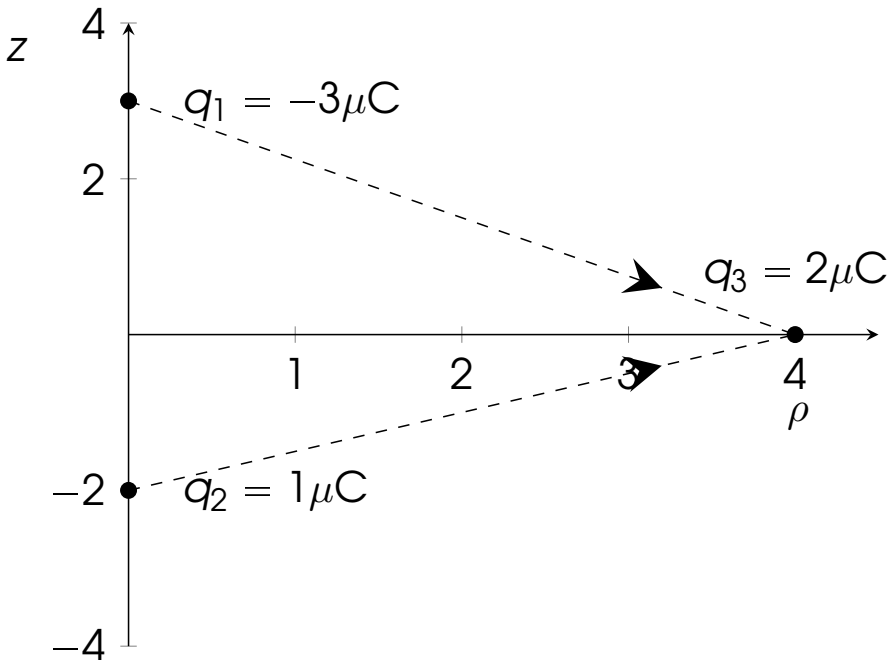
5.1 I can apply Coloumb's Law to use direct integration to calculate the electric field due to charge density in varoius geometries, including rectangular, cylindrical, and spherical coordinate systems.

5.2 I can apply the integral and differential forms of Gauss' Law to calculate the electric field due to charge density in various geometries, including rectangular, cylindrical, and spherical coordinate systems.

5.3 I can calculate the electric field from the scalar electric potential in a variety of geometries.

5.4 I can apply the electric field boundary conditions to calculate electric fields on both sides of a material boundary.

Coloumb Force



$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32}$$

$$\vec{R} = (\hat{\mathbf{x}}4 + \hat{\mathbf{y}}0) - (\hat{\mathbf{x}}0 + \hat{\mathbf{y}}3) = \hat{\mathbf{x}}4 - \hat{\mathbf{y}}3$$

$$\vec{F}_{31} = \frac{(-3\mu C)(2\mu C)}{(4\pi)(8.854 \times 10^{-12})\left(\sqrt{25}\right)^3}(\hat{\mathbf{x}}4 - \hat{\mathbf{y}}3)$$

$$= (-1.724\hat{\mathbf{x}} + 1.293\hat{\mathbf{y}})\,\text{mN}$$

$$\vec{F}_{32} = \frac{(1\mu C)(2\mu C)}{(4\pi)(8.854 \times 10^{-12})\left(\sqrt{20}\right)^3}(\hat{\mathbf{x}}4 + \hat{\mathbf{y}}2)$$

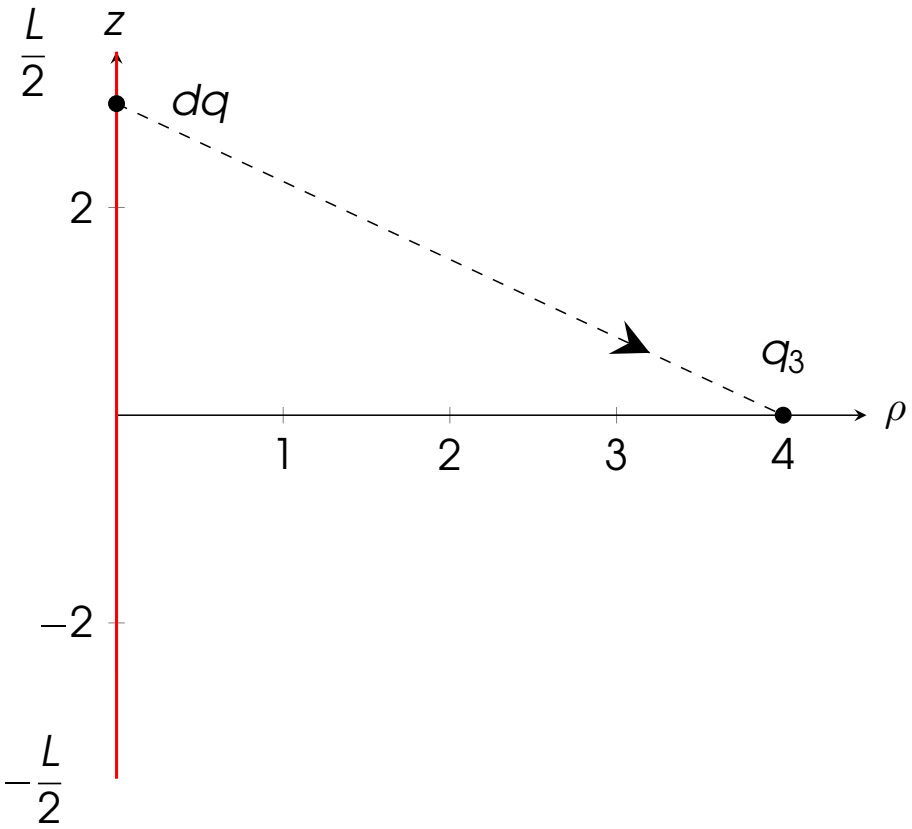
$$= (0.8\hat{\mathbf{x}} + 0.4\hat{\mathbf{y}})\,\text{mN}$$

$$\Longrightarrow \vec{F}_3 = (-0.9\hat{\mathbf{x}} + 1.69\hat{\mathbf{y}})\,\text{mN}$$

$$\vec{F}_{mn} = \frac{q_m q_n}{4\pi|R|^2}\hat{R} = \frac{q_m q_n}{4\pi|R|^3}\vec{R}$$

with: $\vec{R} = \vec{r} - \vec{r}\,'$

$\vec{E}$ by Direct Integration: Line Charge



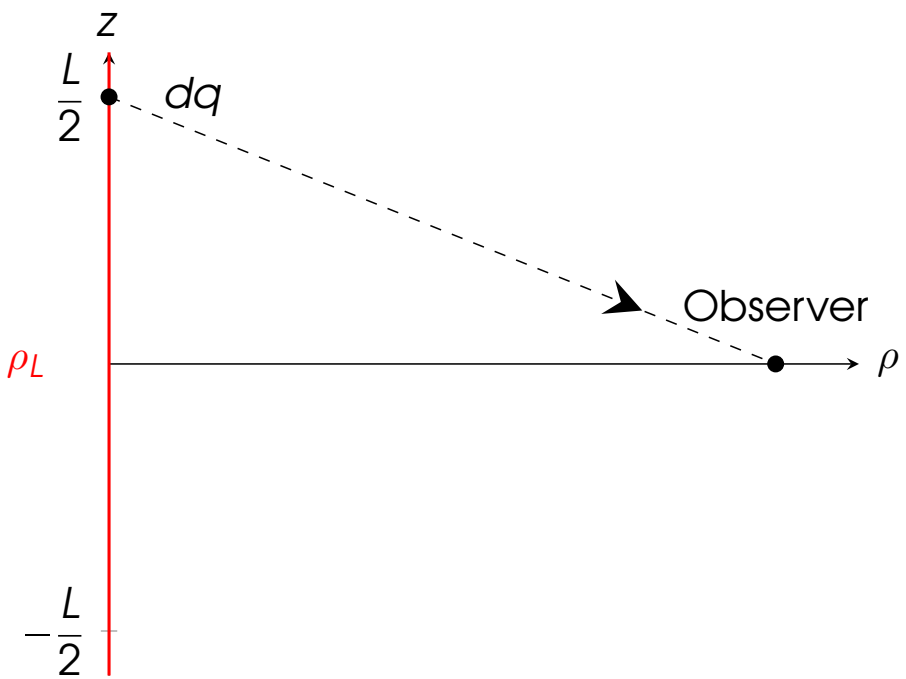
Method

1. Draw picture
2. Write "Coulomb's Law":

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon|R|^3}\vec{R}$$

3. Define dQ in terms of charge distribution (ρ) and length ($d\ell$)
4. Define $\vec{R}$
5. Integrate to find $\vec{E}$

$\vec{E}$ by Direct Integration: Line Charge



Method

1. Draw picture
2. Write "Coulomb's Law":
$$d\vec{E} = \frac{dQ}{4\pi\varepsilon|R|^3}\vec{R}$$
3. Define
$dQ = \rho_\ell d\ell = \rho_L dz$
4. Define $\vec{R} = \hat{\boldsymbol{\rho}}\rho - \hat{\mathbf{z}}z$
$$d\vec{E} = \frac{\rho_L dz}{(4\pi\varepsilon)\left(\sqrt{\rho^2+z^2}\right)^3}$$

$\vec{E}$ by Direct Integration: Line Charge

$\hat{\boldsymbol{\rho}}$ component:

$$\Longrightarrow \hat{\boldsymbol{\rho}}\frac{\rho_L\rho}{4\pi\varepsilon}\int_{-L/2}^{L/2}\frac{1}{\left(\rho^2+z^2\right)^{3/2}}dz$$

$$\Longrightarrow \hat{\boldsymbol{\rho}}\frac{\rho_L\rho}{4\pi\varepsilon}\left(\frac{z}{\rho^2\sqrt{\rho^2+z^2}}\right)\Bigg|_{-L/2}^{L/2}$$

$$= \hat{\boldsymbol{\rho}}\frac{\rho_L\rho}{4\pi\varepsilon}\left(\frac{\frac{L}{2}}{\rho^2\sqrt{\rho^2+\left(\frac{L}{2}\right)^2}}-\frac{-\frac{L}{2}}{\rho^2\sqrt{\rho^2+\left(-\frac{L}{2}\right)^2}}\right)$$

$$= \hat{\boldsymbol{\rho}}\frac{\rho_L\rho}{4\pi\varepsilon}\left(\frac{L}{\rho^2\sqrt{\rho^2+\left(\frac{L}{2}\right)^2}}\right)$$

$$= \hat{\boldsymbol{\rho}}\frac{\rho_L L}{4\pi\varepsilon\rho\sqrt{\rho^2+\left(\frac{L}{2}\right)^2}}$$

$\hat{\mathbf{z}}$ component:

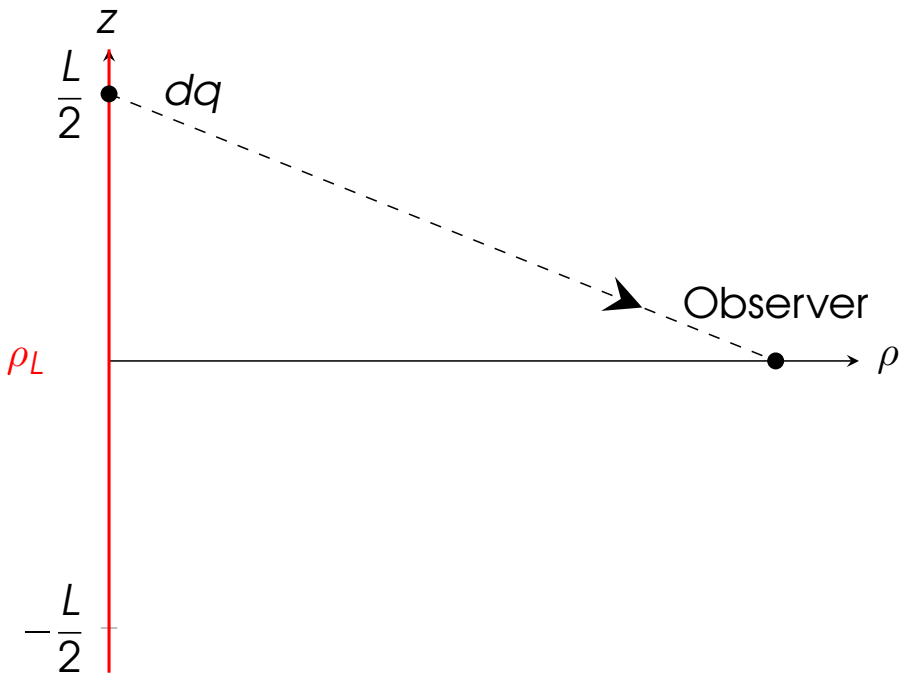
$$\Longrightarrow -\hat{\mathbf{z}}\frac{\rho_L}{4\pi\varepsilon}\int_{-L/2}^{L/2}\frac{z}{\left(\rho^2+z^2\right)^{3/2}}dz$$

$$\Longrightarrow -\hat{\mathbf{z}}\frac{\rho_L}{4\pi\varepsilon}\left(\frac{1}{\sqrt{\rho^2+z^2}}\right)\Bigg|_{-L/2}^{L/2}$$

$$= -\hat{\mathbf{z}}\frac{\rho_L}{4\pi\varepsilon}\left(\frac{1}{\sqrt{\rho^2+\left(\frac{L}{2}\right)^2}}-\frac{1}{\sqrt{\rho^2+\left(-\frac{L}{2}\right)^2}}\right)$$

$$= 0$$

$\vec{E}$ by Direct Integration: Line Charge



Result

$$\vec{E} = \hat{\boldsymbol{\rho}} \frac{\rho_L L}{4\pi\varepsilon\rho\sqrt{\rho^2 + \left(\frac{L}{2}\right)^2}}$$

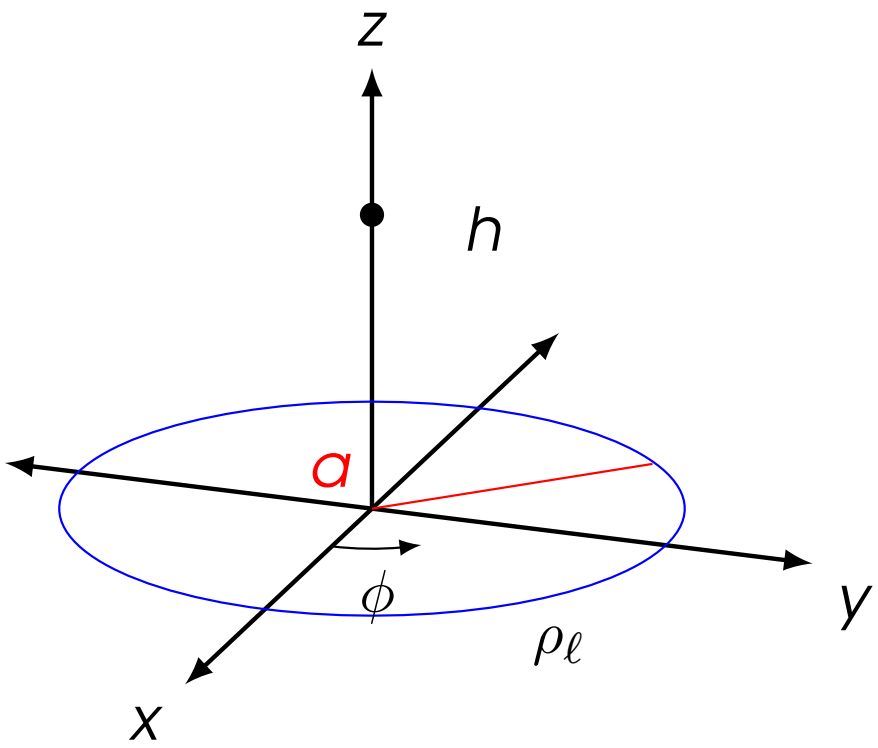
Infinite Wire Approx

For $\frac{L}{2} >> \rho$:

$$\vec{E} = \hat{\boldsymbol{\rho}} \frac{\rho_L L}{\sqrt{(\quad)^2}} = \frac{\rho_L}{2\pi\varepsilon\rho}$$

Wire Loop: Direct Integration

Find the Electric field at $z = h$ from a line charge carrying a charge distribution of ρ_ℓ.



Method

1. Draw picture
2. Write "Coulomb's Law":
$$d\vec{E} = \frac{dQ}{4\pi\varepsilon|R|^3}\vec{R}$$
3. Define
$dQ = \rho_\ell d\ell = \rho_\ell a d\phi$
4. Define $\vec{R} = -a\hat{\boldsymbol{\rho}} + h\hat{\mathbf{z}}$
$$d\vec{E} = \frac{\rho_\ell a}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}}$$
$$(-a\hat{\boldsymbol{\rho}} + h\hat{\mathbf{z}})\, d\phi$$

Wire Loop: Direct Integration (cont'd)

$\hat{\boldsymbol{\rho}}$ component:

$$\Longrightarrow \int_0^{2\pi} \hat{\boldsymbol{\rho}} \frac{\rho_\ell a^2}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} d\phi$$

$$\Longrightarrow \frac{\rho_\ell a^2}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} \int_0^{2\pi} \hat{\boldsymbol{\rho}} d\phi$$

...but $\hat{\boldsymbol{\rho}} = \hat{\mathbf{x}}\cos(\phi) + \hat{\mathbf{y}}\sin(\phi)$

$$\Longrightarrow \frac{\rho_\ell a^2}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} \int_0^{2\pi} \hat{\mathbf{x}}\cos(\phi) + \hat{\mathbf{y}}\sin(\phi)\, d\phi$$

$$\Longrightarrow \frac{\rho_\ell a^2}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} (0+0)$$

$\hat{\mathbf{z}}$ component:

$$\Longrightarrow \int_0^{2\pi} \hat{\mathbf{z}} \frac{\rho_\ell ah}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} d\phi$$

$$\Longrightarrow \hat{\mathbf{z}} \frac{\rho_\ell ah}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} \int_0^{2\pi} d\phi$$

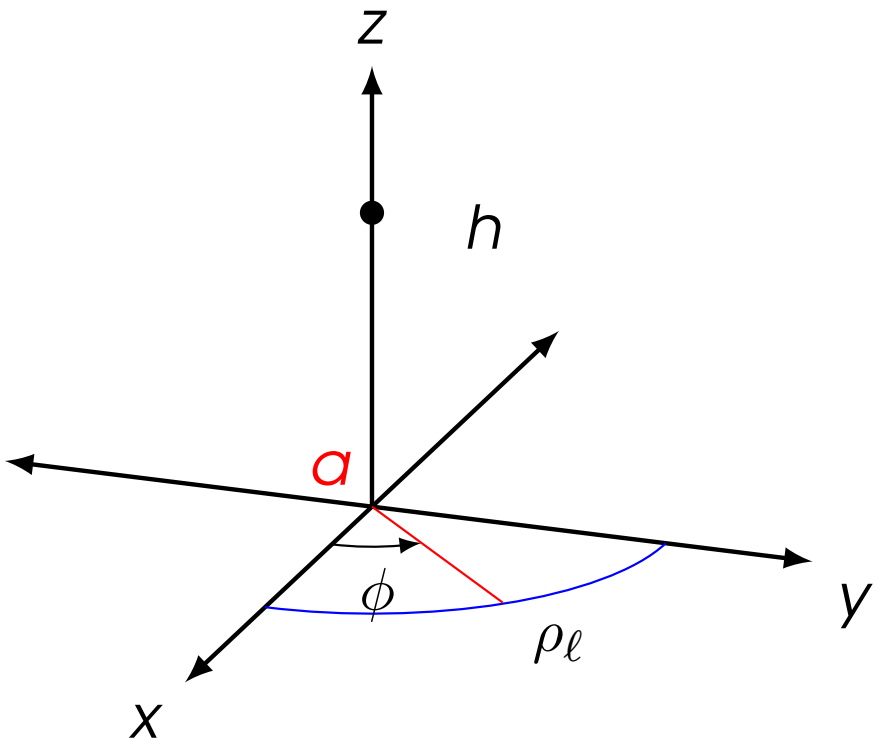
$$\Longrightarrow \hat{\mathbf{z}} \frac{\rho_\ell ah}{2\varepsilon\left(a^2+h^2\right)^{3/2}}$$

Result

$$\vec{E} = \hat{\mathbf{z}} \frac{\rho_\ell ah}{2\varepsilon\left(a^2+h^2\right)^{3/2}}$$

Partial Line: Direct Integration

Find the Electric field at $z = h$ from a partial loop carrying a charge distribution of ρ_ℓ.



Method

1. Draw picture
2. Write "Coulomb's Law":
$$d\vec{E} = \frac{dQ}{4\pi\varepsilon|R|^3}\vec{R}$$
3. Define
$dQ = \rho_\ell \rho d\phi = \rho_\ell a d\phi$
4. Define $\vec{R} = -a\hat{\boldsymbol{\rho}} + h\hat{\mathbf{z}}$
$$d\vec{E} = \frac{\rho_\ell a}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}}$$

Partial Line: Direct Integration

$\hat{\boldsymbol{\rho}}$ component:

$$\Longrightarrow \int_0^{\pi/2} \hat{\boldsymbol{\rho}} \frac{\rho_\ell a^2}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} d\phi$$

$$\Longrightarrow \frac{\rho_\ell a^2}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} \int_0^{\pi/2} \hat{\boldsymbol{\rho}} d\phi$$

...but $\hat{\boldsymbol{\rho}} = \hat{\mathbf{x}}\cos(\phi) + \hat{\mathbf{y}}\sin(\phi)$

$$\Longrightarrow \frac{\rho_\ell a^2}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} \int_0^{\pi/2} \hat{\mathbf{x}}\cos(\phi) + \hat{\mathbf{y}}\sin(\phi)\, d\phi$$

$$\Longrightarrow \frac{\rho_\ell a}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} \left(-\hat{\mathbf{x}}a - \hat{\mathbf{y}}a\right)$$

$\hat{\mathbf{z}}$ component:

$$\Longrightarrow \int_0^{\pi/2} \hat{\mathbf{z}} \frac{\rho_\ell a h}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} d\phi$$

$$\Longrightarrow \hat{\mathbf{z}} \frac{\rho_\ell a}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} \int_0^{\pi/2} h d\phi$$

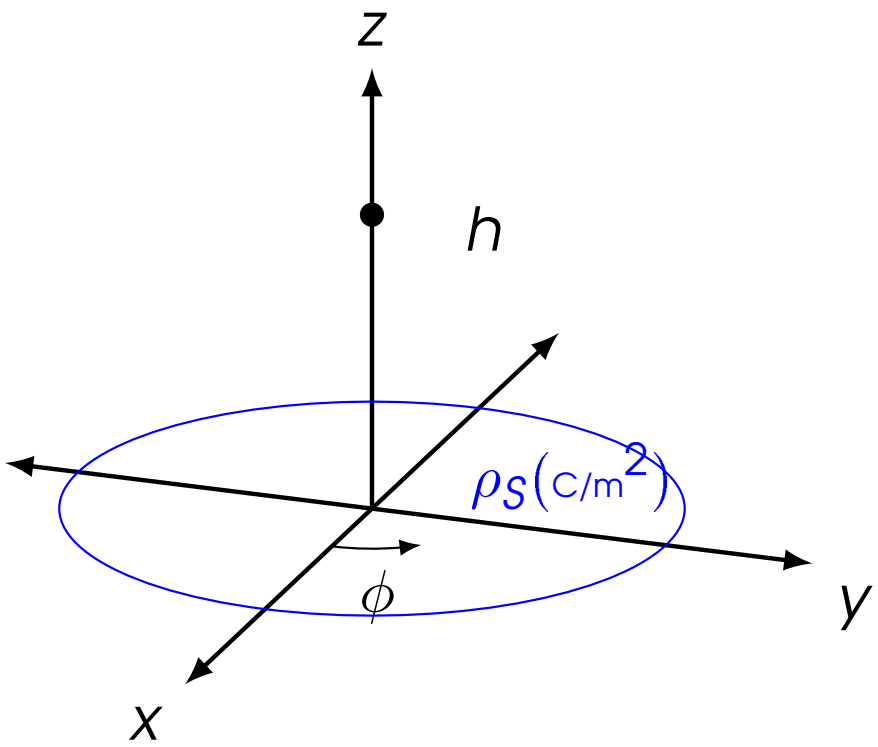
$$\Longrightarrow \hat{\mathbf{z}} \frac{\rho_\ell a}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} \left(\frac{h\pi}{2}\right)$$

Result

$$\vec{E} = \frac{\rho_\ell a}{4\pi\varepsilon\left(a^2+h^2\right)^{3/2}} \left(-\hat{\mathbf{x}}a - \hat{\mathbf{y}}a + \hat{\mathbf{z}}\frac{h\pi}{2}\right)$$

Disk Charge: Direct Integration

Find the Electric field at $z = h$ from a disk carrying a charge distribution of ρ_S.



Method

1. Draw picture
2. Write "Coulomb's Law":

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon|R|^3}\vec{R}$$

3. Define $dQ = \rho_S \rho d\rho d\phi$
4. Define $\vec{R} = -\rho\hat{\boldsymbol{\rho}} + h\hat{\mathbf{z}}$

$$d\vec{E} = \frac{\rho_S \rho}{4\pi\varepsilon\left(\rho^2 + h^2\right)^{3/2}} \left(-\rho\hat{\boldsymbol{\rho}} + h\hat{\mathbf{z}}\right) d\rho d\phi$$

Disk Charge: Direct Integration (cont'd)

6. Integrate to find $\vec{E}$:

$\hat{\rho}$ component $\longrightarrow 0$

$$\hat{\mathbf{z}}\text{ component} \longrightarrow \hat{\mathbf{z}}\frac{\rho_S h}{4\pi\varepsilon}\int_0^{2\pi}\int_0^a \frac{\rho}{\left(\rho^2+h^2\right)^{3/2}}d\rho d\phi = \hat{\mathbf{z}}\frac{\rho_S h}{4\pi\varepsilon}\int_0^{2\pi} d\phi \int_0^a \frac{\rho}{\left(\rho^2+h^2\right)^{3/2}}d\rho$$

$$= \hat{\mathbf{z}}\frac{2\pi\rho_S h}{4\pi\varepsilon}\int_0^a \frac{\rho}{\left(\rho^2+h^2\right)^{3/2}}d\rho = \hat{\mathbf{z}}\frac{\rho_S h}{2\varepsilon}\int_0^a \frac{\rho}{\left(\rho^2+h^2\right)^{3/2}}d\rho$$

$$u=\rho^2+h^2 \qquad du=2\rho d\rho \Longrightarrow \frac{1}{2}\int u^{-3/2}du = -u^{-1/2} \longrightarrow -\frac{1}{\sqrt{\rho^2+h^2}}$$

$$\Longrightarrow -\hat{\mathbf{z}}\frac{\rho_S h}{2\varepsilon\sqrt{\rho^2+h^2}}\Bigg|_{\rho=0}^{a} = \hat{\mathbf{z}}\frac{\rho_S h}{2\varepsilon\sqrt{\rho^2+h^2}}\left(-\frac{1}{\sqrt{a^2+h^2}}+\frac{1}{\sqrt{h^2}}\right)$$

Result

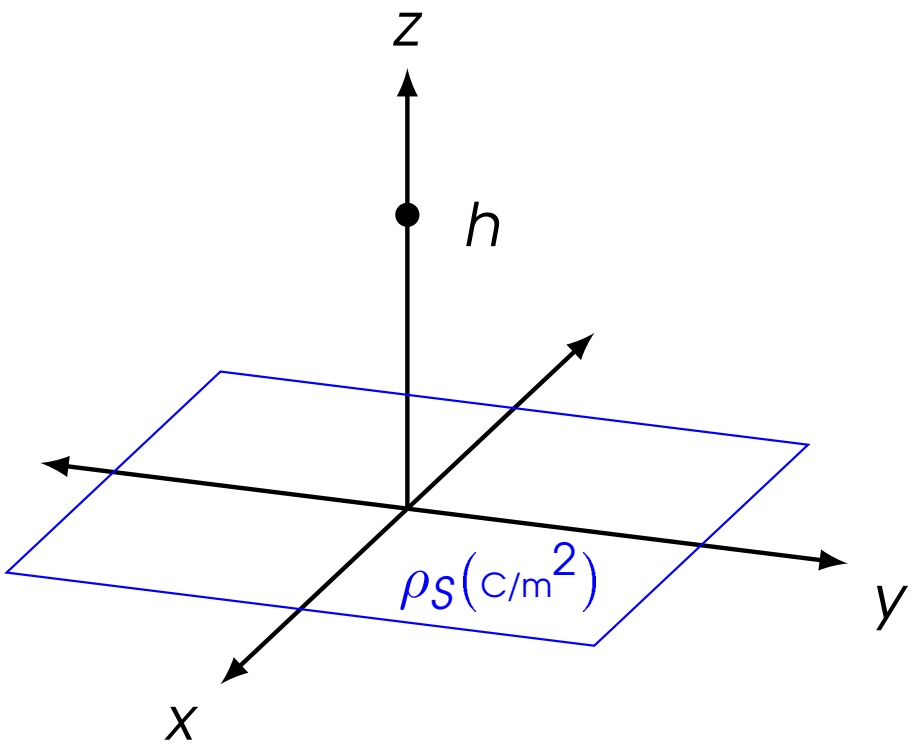
$$\vec{E}=\hat{\mathbf{z}}\frac{\rho_S h}{2\varepsilon}\left(\frac{1}{h}-\frac{1}{\sqrt{a^2+h^2}}\right)$$

Infinite Disk ($h << a$)

$$\vec{E}=\hat{\mathbf{z}}\frac{\rho_S}{2\varepsilon}$$

Sheet Surface Charge: Direct Integration

Find the Electric field at $z = h$ from a sheet carrying a charge distribution of ρ_S.



Method

1. Draw picture
2. Write "Coulomb's Law":
$$d\vec{E} = \frac{dQ}{4\pi\varepsilon|R|^3}\vec{R}$$
3. Define
$dQ = \rho_S dS = \rho_S dx dy$
4. Define $\vec{R} = -\hat{\mathbf{x}}x - \hat{\mathbf{y}}y + \hat{\mathbf{z}}h$

$$d\vec{E} = \frac{\rho_S dx dy}{(4\pi\varepsilon)\left(x^2 + y^2 + h^2\right)^{3/2}}(-\hat{\mathbf{x}}x - \hat{\mathbf{y}}y + \hat{\mathbf{z}}h)$$

Sheet Surface Charge: Direct Integration

Even when we notice the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ components cancel (by symmetry),

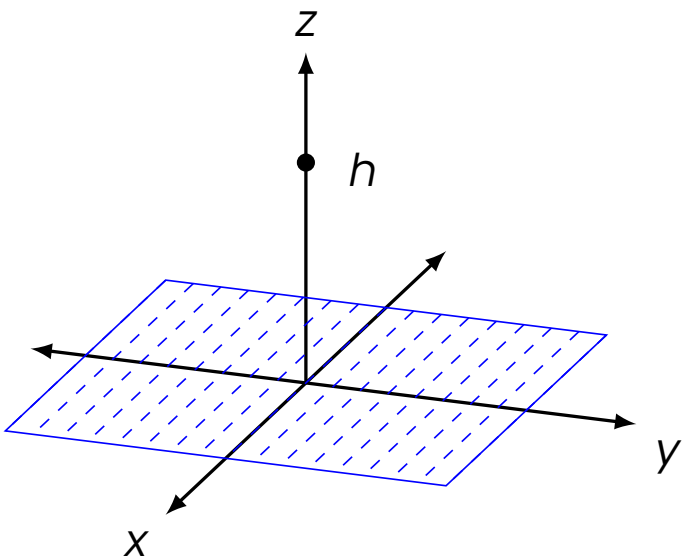
$$\iint \frac{\rho_S}{(4\pi\varepsilon)\left(x^2+y^2+h^2\right)^{3/2}}\left(-\hat{\mathbf{x}}x-\hat{\mathbf{y}}y+\hat{\mathbf{z}}h\right)dxdy \longleftarrow \text{ gross...}$$

Another approach $\longrightarrow$ line charges (with ρ_ℓ) oriented in x-direction...

$$\vec{E}_{\text{line}} = \hat{\boldsymbol{\rho}}\frac{\rho_\ell}{2\pi\varepsilon\rho} \text{ with } \hat{\boldsymbol{\rho}} = \frac{\pm\hat{\mathbf{y}}y+\hat{\mathbf{z}}h}{\sqrt{y^2+h^2}}$$

$$\Longrightarrow \vec{E}_{\text{total}} = \int \vec{E}_{\text{line}}\,dy = \int \hat{\boldsymbol{\rho}}\frac{\rho_\ell}{2\pi\varepsilon\rho}\,dy$$

$$= \frac{\rho_\ell}{2\pi\varepsilon}\int \frac{\pm\hat{\mathbf{y}}t+\hat{\mathbf{z}}h}{\rho^2}\,dy$$



Sheet Surface Charge: Direct Integration

$$= \frac{\rho_\ell}{2\pi\varepsilon}\left(0 + \hat{\mathbf{z}}\int_{-\infty}^{\infty}\frac{h}{y^2+h^2}dy\right)$$

$$= \frac{\rho_\ell h}{2\pi\varepsilon}\left(\hat{\mathbf{z}}\int_{-\infty}^{\infty}\frac{1}{y^2+h^2}dy\right)$$

$$= \hat{\mathbf{z}}\frac{\rho_\ell}{2\pi\varepsilon}\left(\frac{\operatorname{atan}\left(\frac{y}{h}\right)}{h}\right)\Bigg|_{-\infty}^{\infty}$$

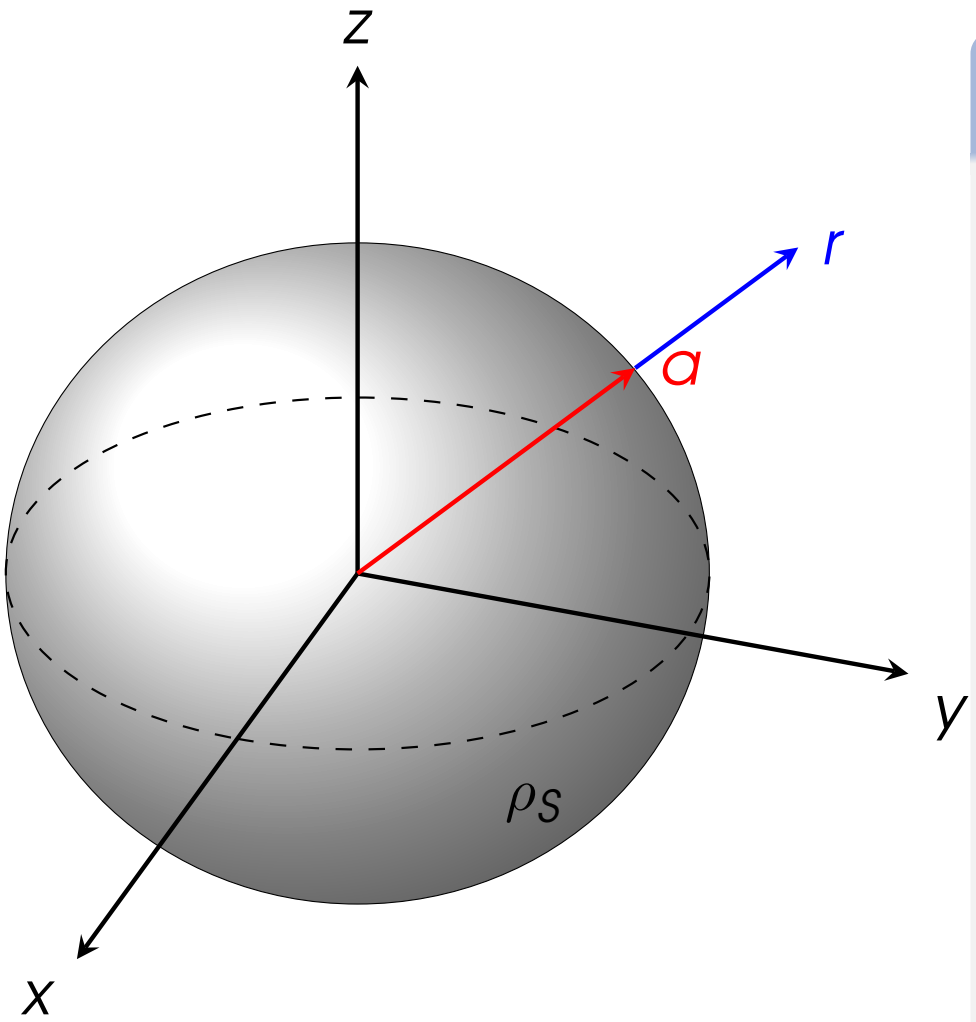
$$= \hat{\mathbf{z}}\frac{\rho_\ell}{2\pi\varepsilon}(\pi)$$

$$\Longrightarrow \vec{E}_{\text{total}} = \hat{\mathbf{z}}\frac{\rho_\ell}{2\varepsilon}$$

Result

$$\vec{E} = \hat{\mathbf{z}}\frac{\rho_\ell}{2\varepsilon}$$

Spherical Surface Charge



Method

1. Draw picture
2. Write "Coulomb's Law":

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon|R|^3}\vec{R}$$

3. Define $dQ = \rho_S dS = \rho_S a^2 \sin(\theta)\, d\theta d\phi$
4. Define $\vec{R} = \hat{\mathbf{r}} r$

$$d\vec{E} = \hat{\mathbf{r}}\frac{\rho_S a^2 \sin(\theta)}{(4\pi\varepsilon)(r^2)} d\theta d\phi$$

Spherical Surface Charge (cont'd)

$$\vec{E} = \int_0^{2\pi}\int_0^{\pi} \hat{\mathbf{r}}\frac{\rho_S a^2 \sin(\theta)}{(4\pi\varepsilon)(r^2)}\,d\theta\,d\phi$$

$$= \hat{\mathbf{r}}\frac{\rho_S a^2}{(4\pi\varepsilon)(r^2)}\int_0^{2\pi} d\phi \int_0^{\pi} \sin(\theta)\,d\theta$$

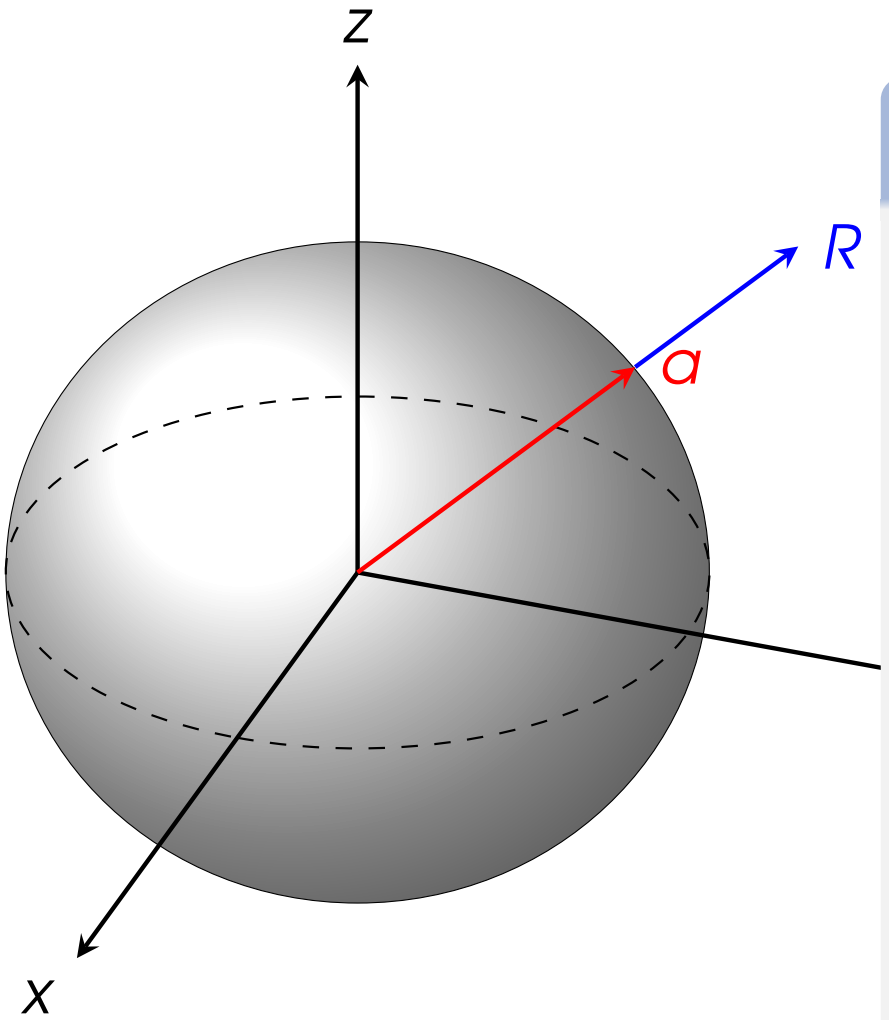
$$= \hat{\mathbf{r}}\frac{\rho_S a^2}{(4\pi\varepsilon)(r^2)}(2\pi)(-\cos(\theta))\Big|_0^{\pi}$$

$$\hat{\mathbf{r}}\frac{\rho_S a^2}{(4\pi\varepsilon)(r^2)}(2\pi)(1+1) = \hat{\mathbf{r}}\frac{\rho_S a^2}{2\varepsilon r^2}$$

Result

$$\vec{E} = \hat{\mathbf{r}}\frac{\rho_S a^2}{\varepsilon r^2}$$

Your Turn: Spherical Volume Charge



Method

1. Draw picture
2. Write "Coulomb's Law":

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon|R|^3}\vec{R}$$

3. Define $dQ = \rho_V dV = \rho_V r^2 \sin(\theta)\, dr d\theta d\phi$
4. Define $\vec{R} = R$

$$d\vec{E} = \hat{\mathbf{r}}\frac{\rho_V r^2}{4\pi\varepsilon R^3}\sin(\theta)\, dr d\theta d\phi$$

Spherical Volume Charge (cont'd)

$$\begin{aligned}\vec{E} &= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \hat{\mathbf{r}}\frac{\rho_V r^2}{4\pi\varepsilon R^3}\sin(\theta)\, dr d\theta d\phi \\ &= \hat{\mathbf{r}}\frac{\rho_V}{4\pi\varepsilon R^3}\int_0^{2\pi} d\phi \int_0^{\pi}\sin(\theta)\, d\theta \int_0^{a} r^2 dr \\ &= \hat{\mathbf{r}}\frac{\rho_V a^3}{(3)4\pi\varepsilon R^3}(2\pi)(2)\end{aligned}$$

Result

$$\vec{E} = \hat{\mathbf{r}}\frac{\rho_V a^3}{3\varepsilon R^3}$$