ECE343 Spring 2025

Objective 4.13:
Vector Analysis



UNITED STATES
AIR FORCE
ACADEMY

Lesson	Topics	Reading	Notes
1	Course Overview & Introduction	Ch.1	LO 1.1-1.2
Chapter 2: Electric and Magnetic Fields			
2	Fields, Parameters, Field Quantities	Ch. 2	LO 2.1-2.2
Chapter 3: Transmission Lines			
3	Lumped Element Model	3.1-3.4	LO 3.1
4	Telegrapher's Equations	3.5-3.6	LO 3.1
5	Characteristic Impedance & Wave Propagation	3.7-3.9	LO 3.2
6	Coaxial, Reflection Coefficient, VSWR	3.10,3.12-3.14	LO 3.2-3.3
7	Terminated Transmission Lines	3.15,3.17-3.19	LO 3.4
8	Impedance Matching	3.21-3.23	LO 3.4
9	Impedance Matching	3.21-3.23	LO 3.4
10	Smith Charts	Supplemental	LO 3.5
11	Smith Charts	Supplemental	LO 3.5
12	Smith Charts	Supplemental	LO 3.5
13	Smith Charts	Supplemental	LO 3.5
14	Transients	Supplemental	LO 3.6
15	Flex Day		
16	Reflection/Transmission Coefficient Lab	Lab Packet	LO 3.7
17	Reflection/Transmission Coefficient Lab	Lab Packet	LO 3.7
Chapter 4: Vector Analysis			
18	Vector & Coordinate System Review	Ch. 4	LO 4.1-4.4 **GR 1 due**
Chapter 5: Electrostatics			
19	Coulomb's Law	5.1-5.3	LO 5.1
20	Surface and Volume Charge	5.4	LO 5.1
21	Gauss' Law (Integral Form)	5.5-5.6	LO 5.2
22	Gauss' Law (Differential Form)	5.7-5.8	LO 5.2
23	Electric Potential	5.9-5.10, 5.15	LO 5.3

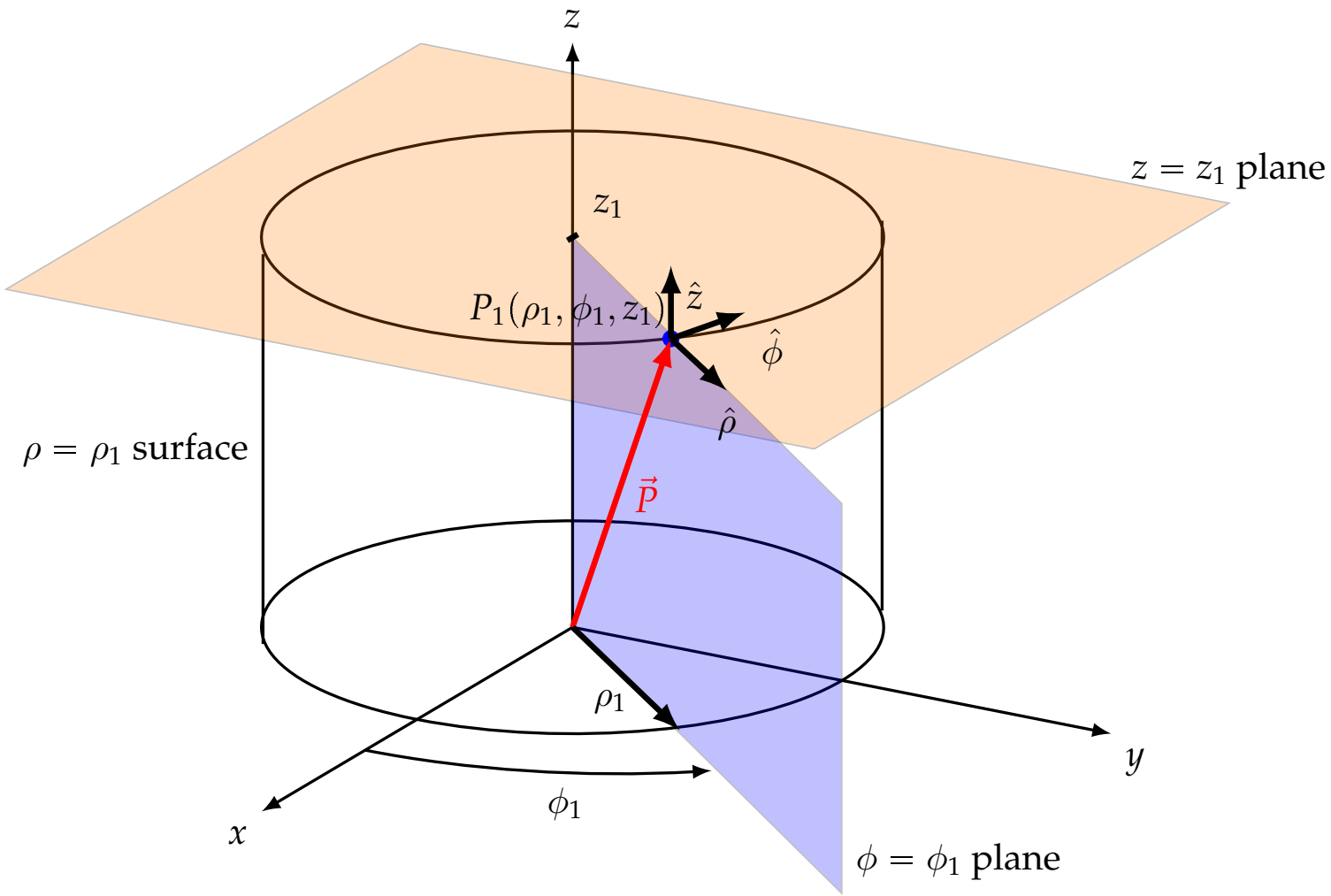
Ch 4 Learning Objectives

4.1 I can recognize and apply common vector calculus operations, such as: gradient, divergence, curl, and Laplacian in the three major coordinate systems.

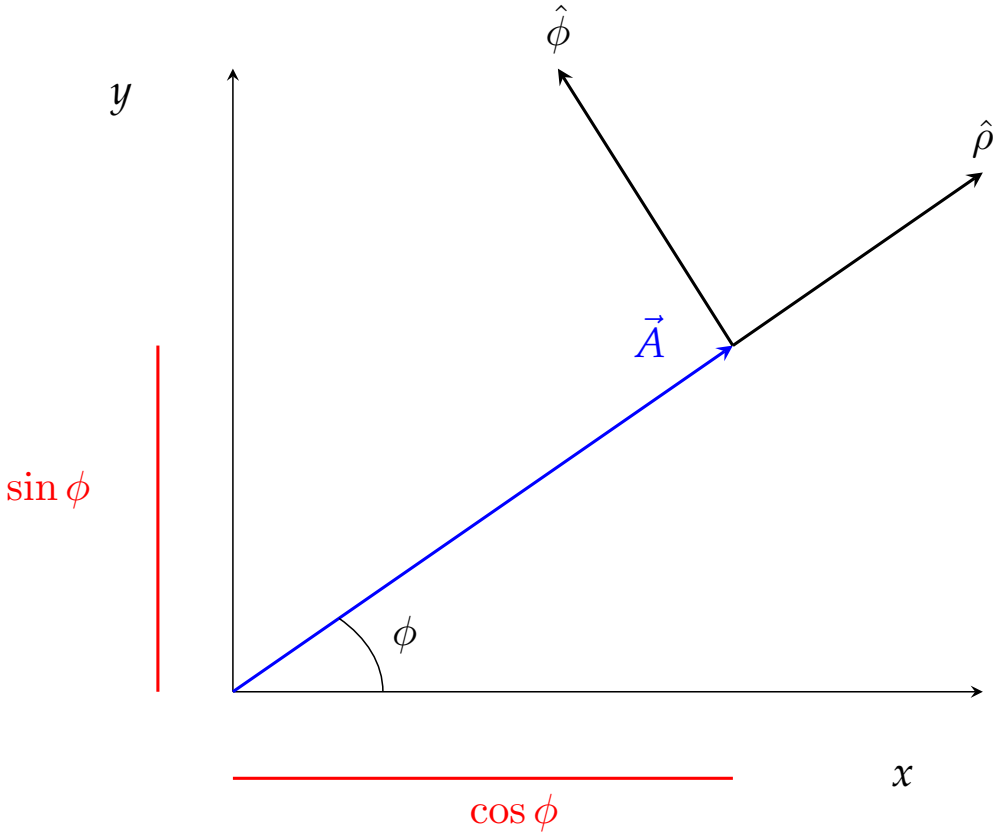
4.2 I can calculate surface and volume integrals in rectangular, cylindrical, and spherical coordinate systems.

4.3 I can differentiate between Stoke's Theorem and the Divergence Theorem and apply them to various problems to simplify integrals.

Cylindrical Coords - Review



Cylindrical Coords - Transformation



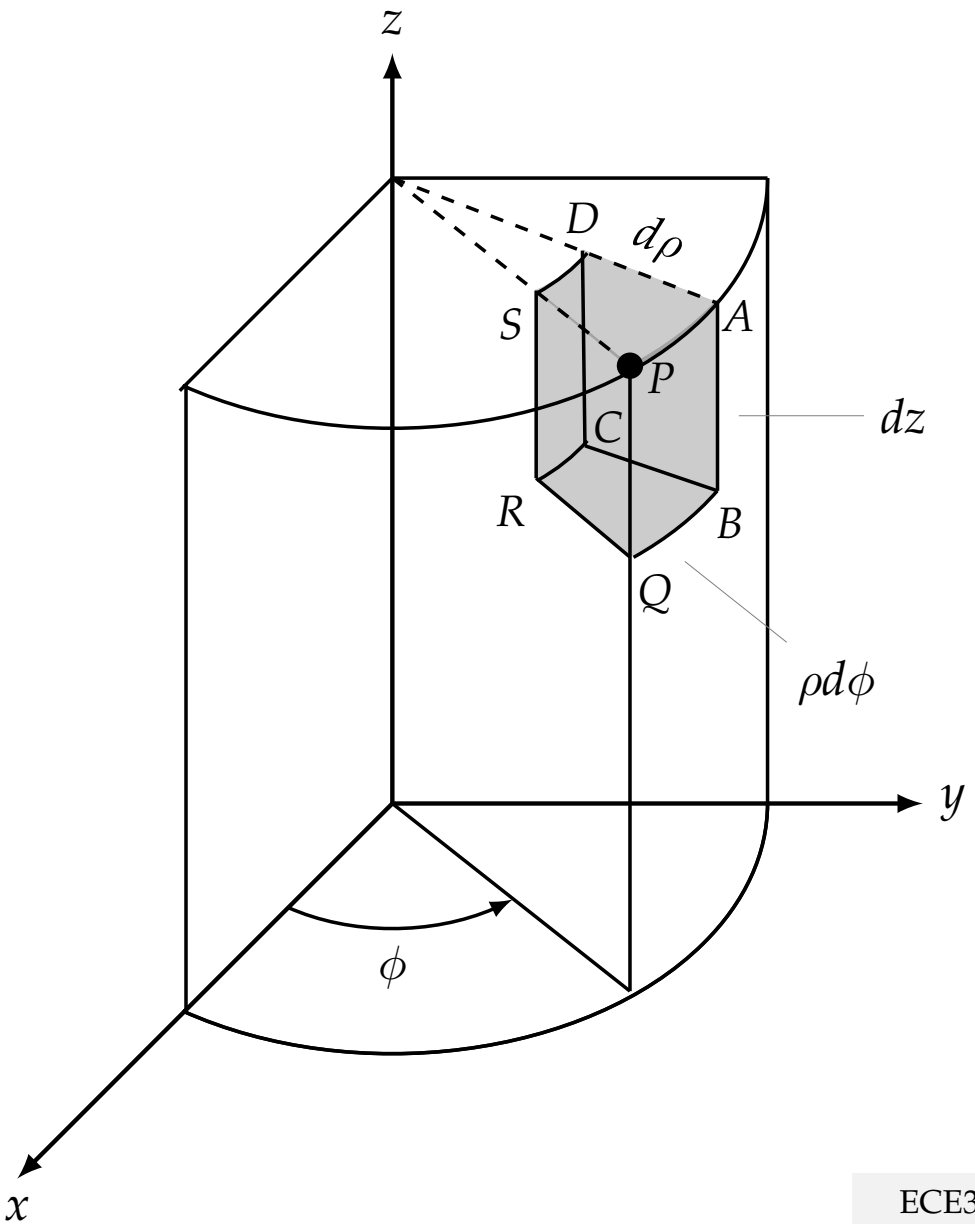
$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$
$$z = z$$

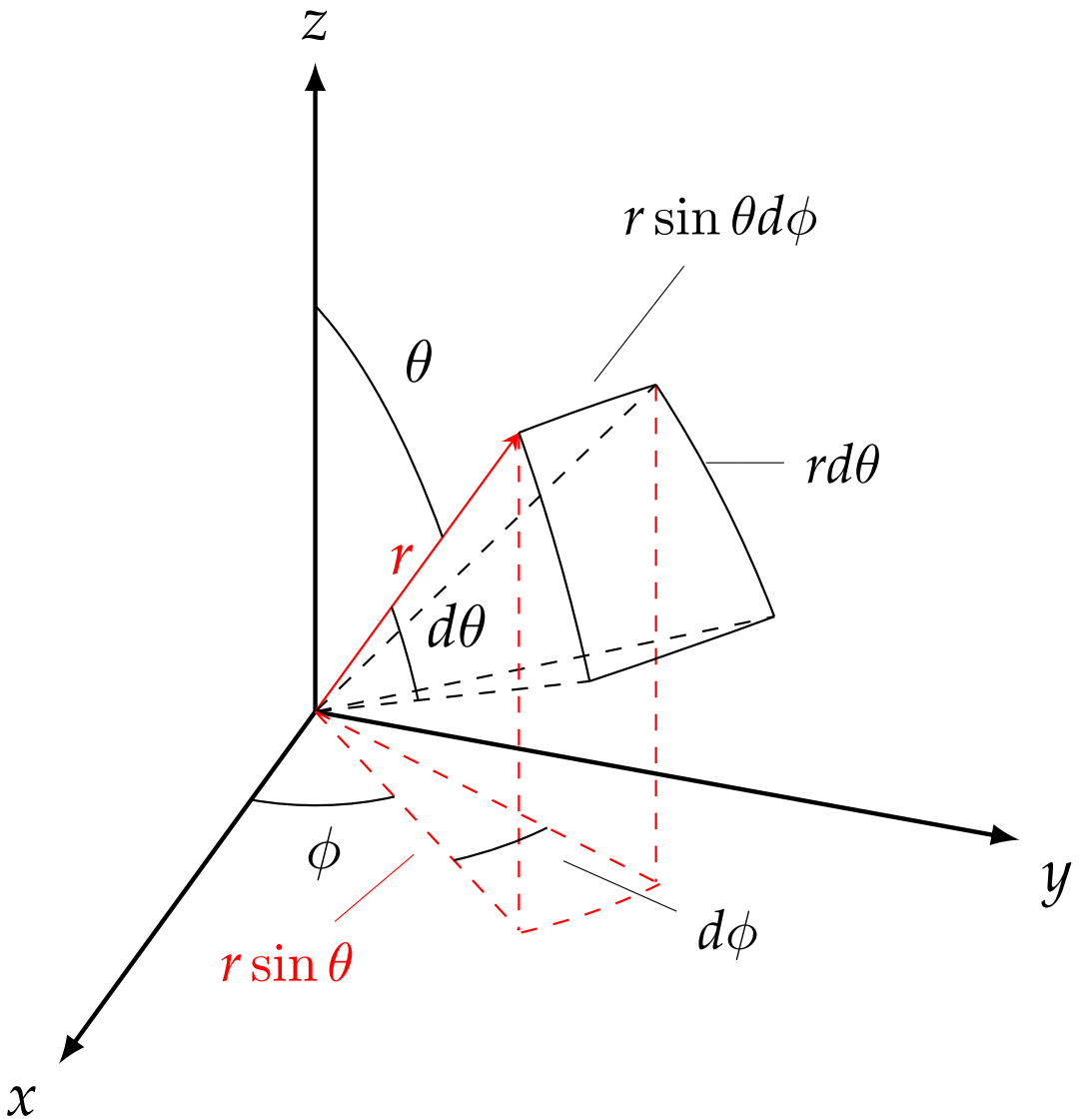
$$\hat{\boldsymbol{\rho}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$$
$$\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$$
$$\hat{\mathbf{z}} = \hat{\mathbf{z}}$$

$$\hat{\mathbf{x}} = \hat{\boldsymbol{\rho}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$$
$$\hat{\mathbf{y}} = \hat{\boldsymbol{\rho}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$$
$$\hat{\mathbf{z}} = \hat{\mathbf{z}}$$

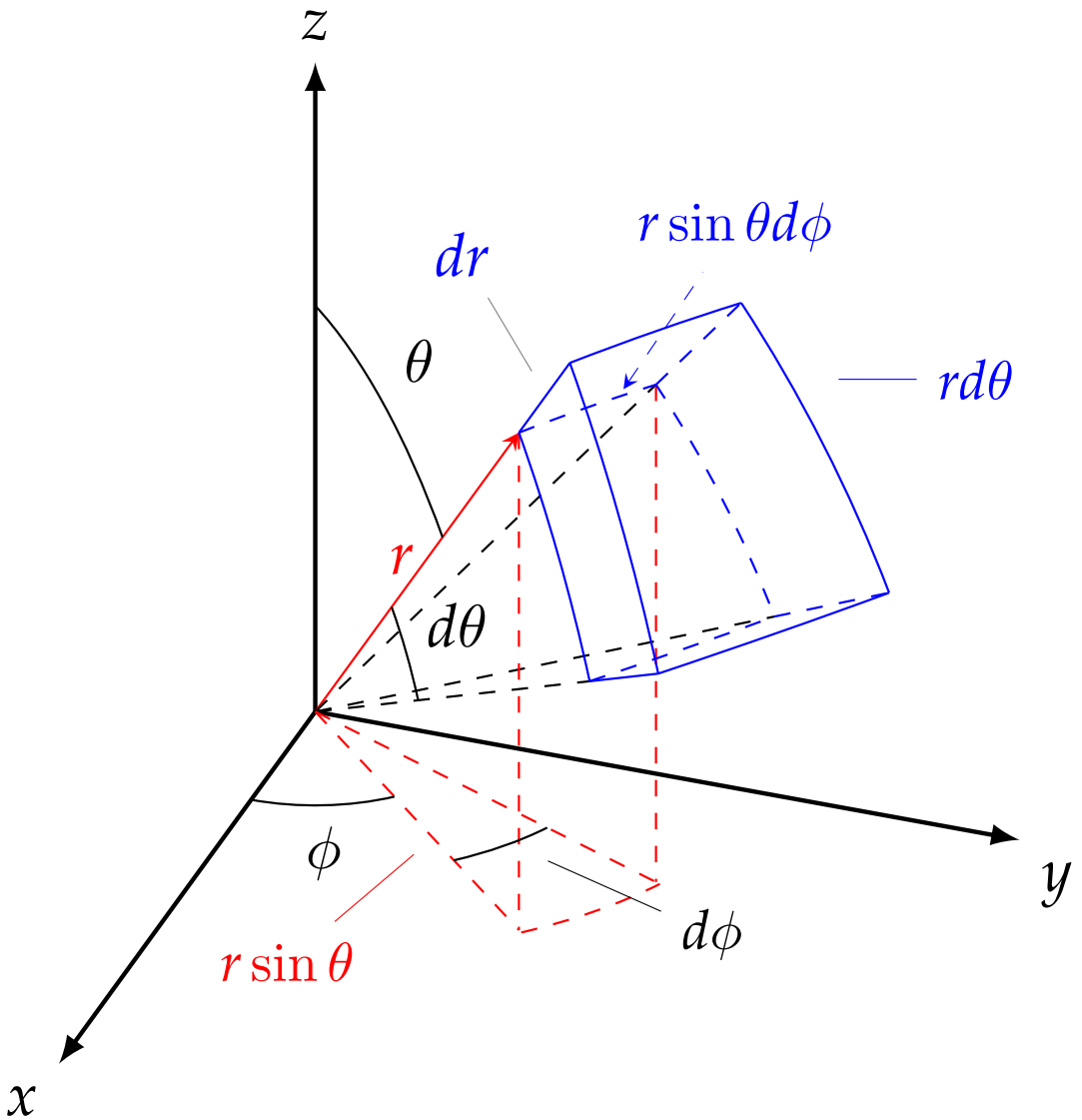
Cylindrcial Differential Elements



Spherical Differential Elements



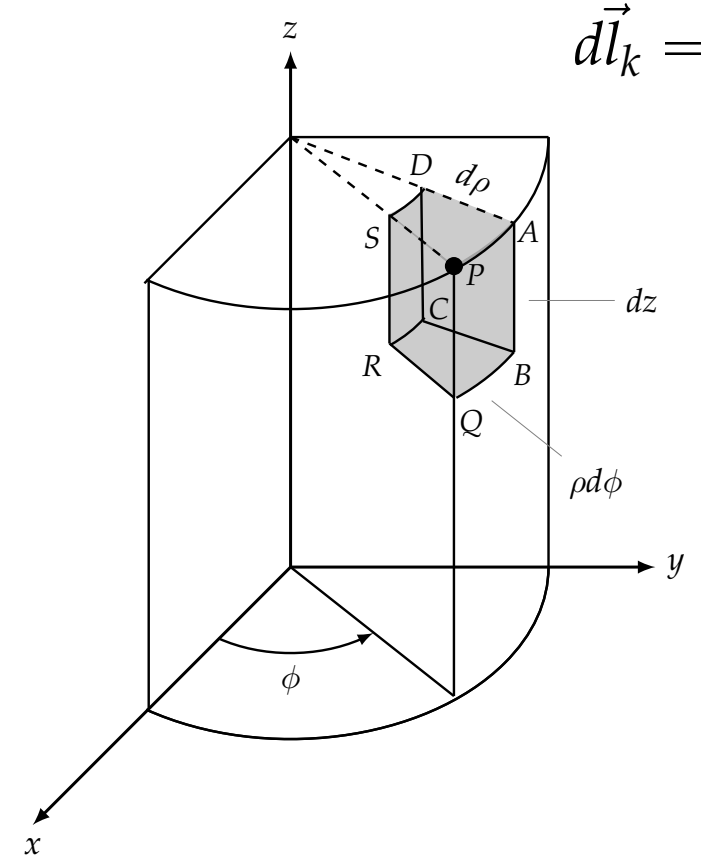
Spherical Differential Elements - Volume



Generalized Coordinate System

$$\vec{A} = \hat{u}_1 A_1 + \hat{u}_2 A_2 + \hat{u}_3 A_3$$

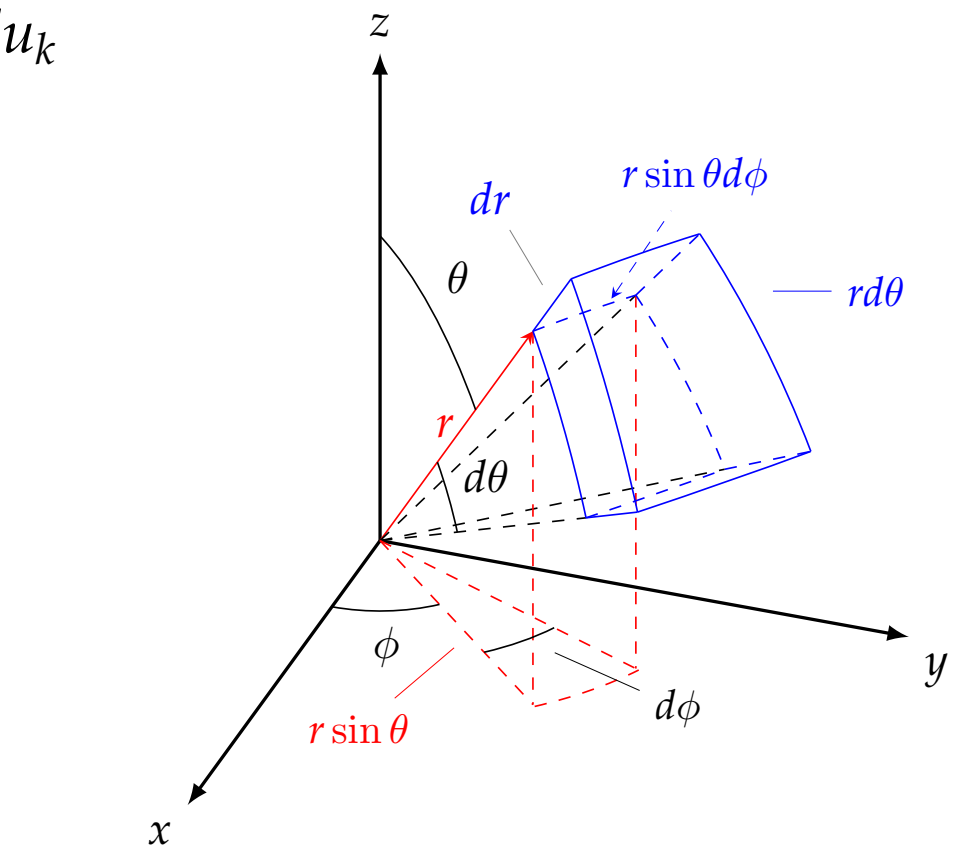
$$d\vec{l}_k = \hat{u}_k h_k du_k$$



$u_1 \longrightarrow$

$u_2 \longrightarrow$

$u_3 \longrightarrow$



$u_1 \longrightarrow$

$u_2 \longrightarrow$

$u_3 \longrightarrow$

Nabla (del) Operator

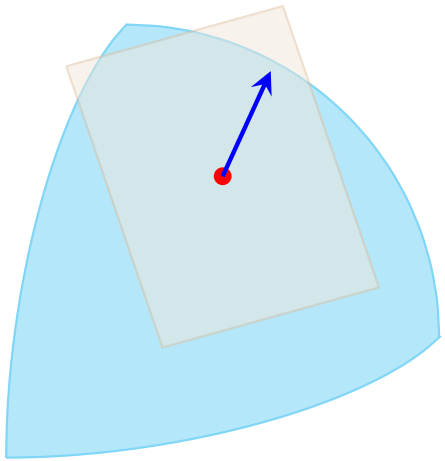
$$\nabla = \hat{u}_1 \frac{1}{h_1} \frac{\partial}{\partial u_1} + \hat{u}_2 \frac{1}{h_2} \frac{\partial}{\partial u_2} + \hat{u}_3 \frac{1}{h_3} \frac{\partial}{\partial u_3}$$

- Rectangular $\longrightarrow$
- Cylindrical $\longrightarrow$
- Spherical $\longrightarrow$

Gradient

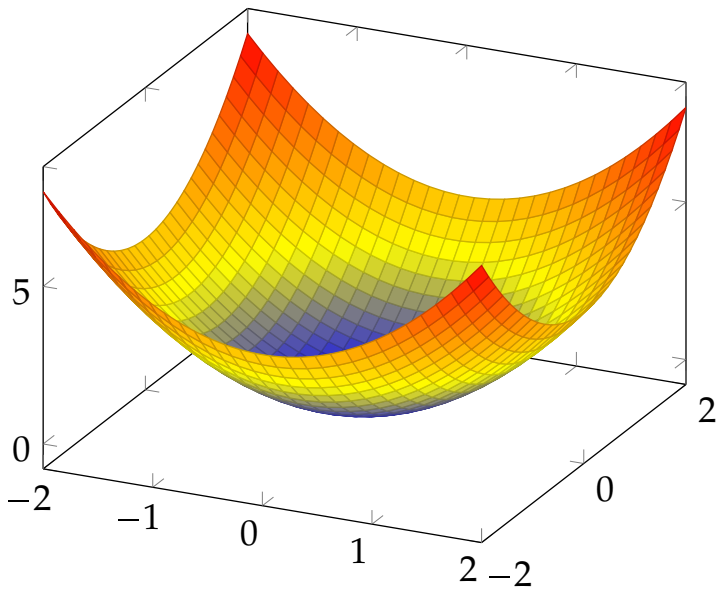
$$\nabla f = \hat{u}_1 \frac{1}{h_1} \frac{\partial f}{\partial u_1} + \hat{u}_2 \frac{1}{h_2} \frac{\partial f}{\partial u_2} + \hat{u}_3 \frac{1}{h_3} \frac{\partial f}{\partial u_3}$$

- $\nabla f(a,b)$ points in the direction of greatest increase of f at (a,b)
- $|\nabla f(a,b)|$ is the maximum rate of change of f at (a,b)
- ∇f is perpendicular to the level curves of f (two variable) or level surfaces of f (three variables)

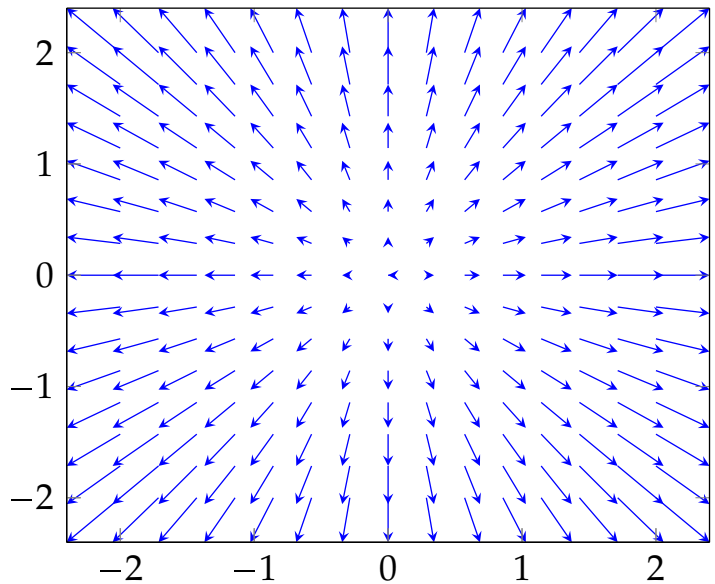


Gradient - Examples

$f(x) = x^2 + y^2$



∇f



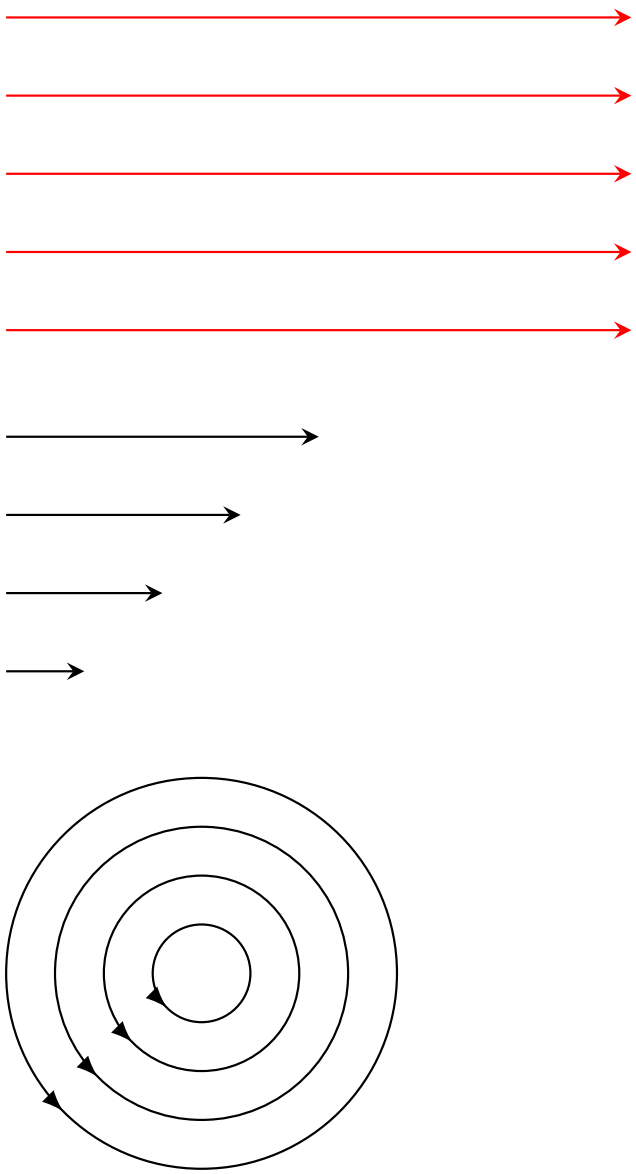
Gradient - Examples

- Find the gradient of $f(x, y, z) = x^4 + y^4$
- Find the gradient of $f(x, y, z) = xy$ at the point (-4,5)

Curl

$$\nabla \times F = \begin{vmatrix} \frac{\hat{u_1}}{h_2 h_3} & \frac{\hat{u_2}}{h_1 h_3} & \frac{\hat{u_3}}{h_1 h_2} \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 F_1 & h_2 F_2 & h_3 F_3 \end{vmatrix}$$

Curl - Examples



Determine the rotation in the field:

$$\vec{H} = \hat{\mathbf{x}}\left(3y + \frac{z^2}{2}\right) + \hat{\mathbf{y}}\left(2x - z\right) + \hat{\mathbf{z}}3xy$$

Divergence

$$\nabla \cdot f = \frac{1}{h_1 h_2 h_3}\left[\frac{1}{h_1}\frac{\partial h_2 h_3 f_1}{\partial u_1} + \frac{1}{h_2}\frac{\partial h_1 h_3 f_2}{\partial u_2} + \frac{1}{h_3}\frac{\partial h_1 h_2 f_3}{\partial u_3}\right]$$

Divergence - Example

Determine the electric flux through a point (x, y, z) of the field:

$$\vec{H} = \hat{\mathbf{x}}\left(3y + \frac{z^2}{2}\right) + \hat{\mathbf{y}}\left(2x - z\right) + \hat{\mathbf{z}}3xy$$

What is Divergence, really?

- Amount of a field "leaving" an enclosed surface:

$$\nabla \cdot \vec{F} = \lim_{\Delta v \to 0} \frac{\oiint_S \vec{F} \cdot d\vec{S}}{\Delta v}$$

- Gauss' Law:

$$\oiint_S \vec{D} \cdot d\vec{S} = \oiint_S \epsilon \vec{E} \cdot d\vec{S} = Q_{\text{enclosed}} \qquad \text{and} \qquad \nabla \cdot \vec{D} = \rho_v$$

Laplacian

$$\nabla^2 V(x,y,z) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V(\rho,\phi,z) = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)$$

$$+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin\theta^2}\frac{\partial^2 V}{\partial \phi^2}$$

Laplacian Examples

- Find the Laplacian of $f(x, y, z) = e^{xyz}$
- Find the Laplacian of $f(x, y, z) = \dfrac{1}{x^2 + y^2 + z^2}$

Divergence Theorem & Stoke's Theorem

Divergence Theorem

$$\iiint_V \nabla \cdot \vec{D} dV = \oiint_S \vec{D} \cdot d\vec{S}$$

Stoke's Theorem

$$\iint_S (\nabla \times F) \cdot dS = \oint_C F \cdot d\ell$$

Stoke's Theorem

Verify the contour integral, using Stoke's theorem:
$\vec{E} = \hat{\rho}\rho\sin\phi + \hat{\phi}\rho^2$ over the contour
$(0,0,0) - (2,0,0) - (2,\frac{\pi}{2},0) - (0,\frac{\pi}{2},0)$