ECE343 Spring 2025

Objective 3.4: Impedance Matching



Lesson	Topics	Reading	Notes
1	Course Overview & Introduction	Ch.1	LO 1.1-1.2
Chapter 2: Electric and Magnetic Fields			
2	Fields, Parameters, Field Quantities	Ch. 2	LO 2.1-2.2
Chapter 3: Transmission Lines			
3	Lumped Element Model	3.1-3.4	LO 3.1
4	Telegrapher's Equations	3.5-3.6	LO 3.1
5	Characteristic Impedance & Wave Propagation	3.7-3.9	LO 3.2
6	Coaxial, Reflection Coefficient, VSWR	3.10,3.12-3.14	LO 3.2-3.3
7	Terminated Transmission Lines	3.15,3.17-3.19	LO 3.4
8	Impedance Matching	3.21-3.23	LO 3.4
9	Impedance Matching	3.21-3.23	LO 3.4
10	Smith Charts	Supplemental	LO 3.5
11	Smith Charts	Supplemental	LO 3.5
12	Smith Charts	Supplemental	LO 3.5
13	Smith Charts	Supplemental	LO 3.5
14	Transients	Supplemental	LO 3.6
15	Reflection/Transmission Coefficient Lab	Lab Packet	LO 3.7
Chapter 4: Vector Analysis			
16	Vector & Coordinate System Review	Ch. 4	LO 4.1-4.4 **GR 1 due**
Chapter 5: Electrostatics			
17	Coulomb's Law	5.1-5.3	LO 5.1
18	Surface and Volume Charge	5.4	LO 5.1
19	Gauss' Law (Integral Form)	5.5-5.6	LO 5.2
20	Gauss' Law (Differential Form)	5.7-5.8	LO 5.2
21	Electric Potential	5.9-5.10, 5.15	LO 5.3

Ch 3 Learning Objectives

3.1 I can describe the origin of the transmission line equation, understand the method of developing a solution, and interpret the solution.

3.2 I can explain the physical meaning of and calculate the following quantities: propagation constant, phase constant, attenuation constant, characteristic impedance, VSWR, reflection coefficient, and transmission coefficient.

3.3 I can use the transmission line equation to calculate the voltage/current at an arbitrary point on a specified transmission line.

3.4 I can use the transmission line equation to match a load to a source using a length of transmission line.

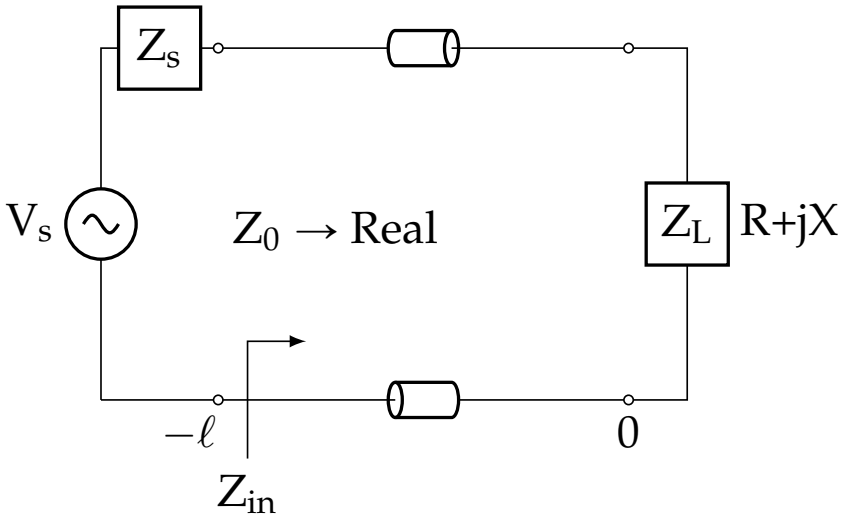
3.5 I can identify the various quantities represented on the Smith Chart.

3.6 I can use a Smith Chart to design a matching network for a given source and load, using series components, parallel components, shorted stubs, and open stubs.

3.7 I can plot the transient response of a load connected to a source through a transmission line.

3.8 I can measure the reflection coefficient across a range of frequencies and interpret the measurement using a network analyzer.

Tx Line Equation



Goal

Match Z_s to Z_{in}, so all P_s is delivered to the load.

Recall the solutions to the voltage and current wave equations:

$$\tilde{V} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$\tilde{I} = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

With, $\Gamma(z) = |\Gamma_L| e^{2\gamma z} = \dfrac{V_0^-}{V_0^+} e^{2\gamma z}$, we can write the **lossless case** as:

$$\tilde{V} = V_0^+ e^{-j\beta z} \left(1 + |\Gamma_L| e^{j2\beta z}\right)$$
$$\tilde{I} = I_0^+ e^{-j\beta z} \left(1 + |\Gamma_L| e^{j2\beta z}\right)$$

Tx Line Equation

Now, we can express the **input impedance** (Z_{in}), at $z = -\ell$ as:

$$Z_{in} = \frac{\widetilde{V}}{\widetilde{I}} = \frac{V_0^+ e^{-j\beta z}\left(1 + |\Gamma_L| e^{j2\beta z}\right)}{I_0^+ e^{-j\beta z}\left(1 + |\Gamma_L| e^{j2\beta z}\right)} = Z_0\left[\frac{1 + \Gamma_L e^{-j2\beta\ell}}{1 - \Gamma_L e^{-j2\beta\ell}}\right]$$

$$= Z_0\left[\frac{1 + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j2\beta\ell}}{1 - \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j2\beta\ell}}\right]$$

$$= Z_0\left[\frac{(Z_L + Z_0)\, e^{+j\beta\ell} + (Z_L - Z_0)\, e^{-j\beta\ell}}{(Z_L + Z_0)\, e^{+j\beta\ell} - (Z_L - Z_0)\, e^{-j\beta\ell}}\right]$$

$$= Z_0\left[\frac{Z_L 2\cos(\beta\ell) + jZ_0 2\sin(\beta\ell)}{Z_0 2\cos(\beta\ell) + jZ_L 2\sin(\beta\ell)}\right]$$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan(\beta\ell)}{Z_0 + jZ_L\tan(\beta\ell)}\right]$$

Tx Line Equation

General

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tanh(\gamma \ell)}{Z_0 + jZ_L \tanh(\gamma \ell)} \right]$$

$$Z_{in} = \frac{z_L + j\tanh(\gamma \ell)}{1 + jz_L \tanh(\gamma \ell)}$$

$$z_L = \frac{Z_L}{Z_0}$$

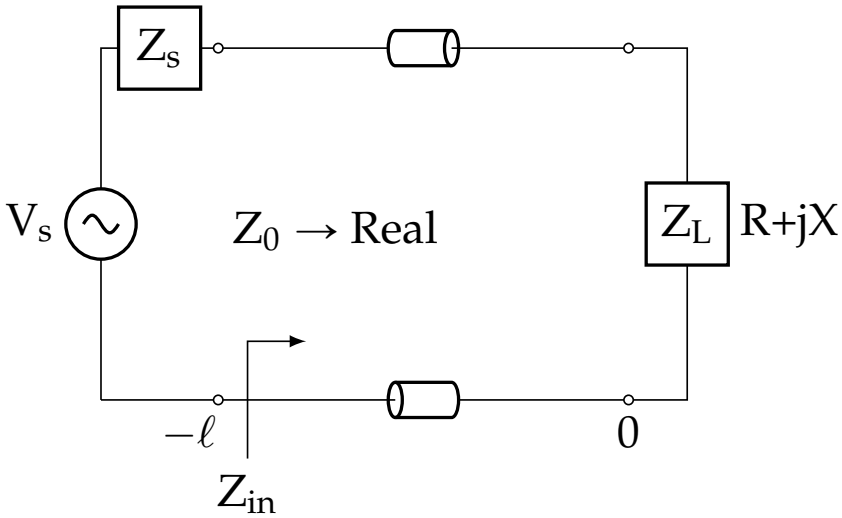
Lossless

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta \ell)}{Z_0 + jZ_L \tan(\beta \ell)} \right]$$

$$Z_{in} = Z_0 \left[\frac{z_L + j\tan(\beta \ell)}{1 + jz_L \tan(\beta \ell)} \right]$$

$$z_L = \frac{Z_L}{Z_0}$$

Tx Line Equation Example: $\ell = \lambda/2$



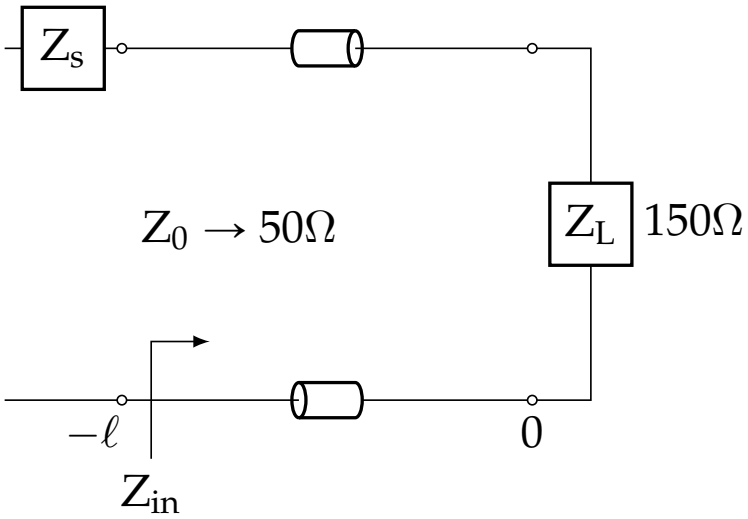
Result

A tx line with a length of $\frac{n\lambda}{2}$ is "transparent".

$$\begin{aligned}\beta &= \frac{2\pi}{\lambda} \text{ and } \beta\ell = \pi \\ &= \left[Z_0 \frac{z_L + j\tan(\pi)}{1 + jz_L\tan(\pi)}\right] \\ &= \frac{z_L}{1}\end{aligned}$$

Tx Line Equation: Example #2

Find Γ_L, VSWR, and $Z_{in}(z = -\frac{\lambda}{8})$:



(1) $\Gamma_L = \dfrac{z_L - 1}{z_L + 1} = \dfrac{3-1}{3+1} = 0.5$

(2) $\text{VSWR} = \dfrac{1 + |\Gamma_L}{1 - \Gamma_L} = \dfrac{1.5}{0.5} = 3$

(3) $Z_{in}(z = -\frac{\lambda}{8})$

$$\ell = \frac{\lambda}{8} \implies \beta\ell = \frac{\pi}{4}$$

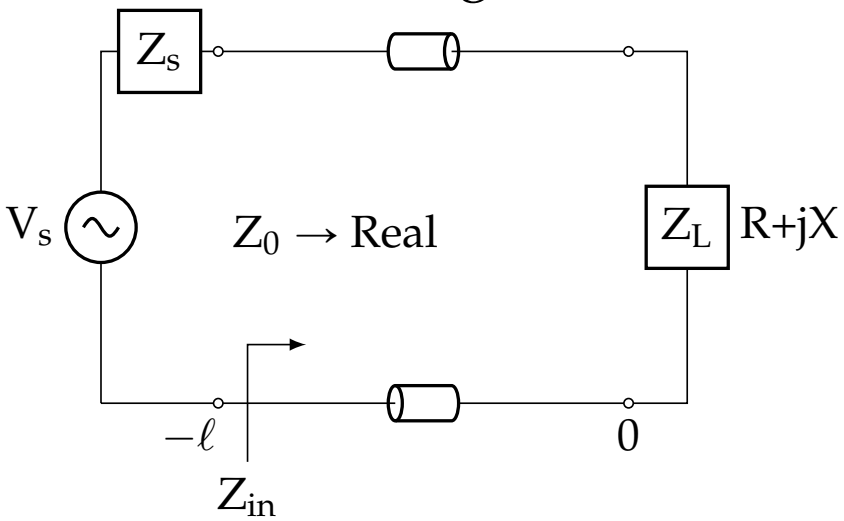
$$Z_{in} = \frac{z_L + j\tan\left(\frac{\pi}{4}\right)}{1 + jz_L\tan\left(\frac{\pi}{4}\right)}$$

$$= 50\Omega\left[\frac{3+j}{1+j3}\right]$$

$$= 30 - j40\Omega$$

Impedance Matching

Now, back to our goal of matching load (Z_L and Tx Line) to Z_s...



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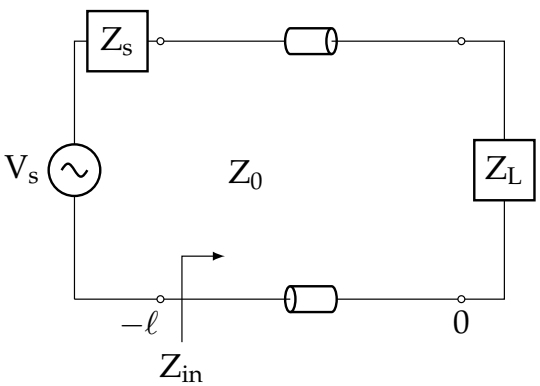
$$Z_{in} = \frac{z_L + j\tan\left(\frac{\pi}{4}\right)}{1 + jz_L\tan\left(\frac{\pi}{4}\right)}$$

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Impedance Matching

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Process

1. Normalize impedances to Z_0
2. Find reflection coefficient (Γ_L)
3. Holding the magnitude of Γ_L constant, find the input impedance $z_{\text{in}} = 1 + jx$

$$\implies |\Gamma_L| = \frac{z_{\text{in}} - 1}{z_{\text{in}} + 1}$$

4. Determine capacitance or inductance from X.

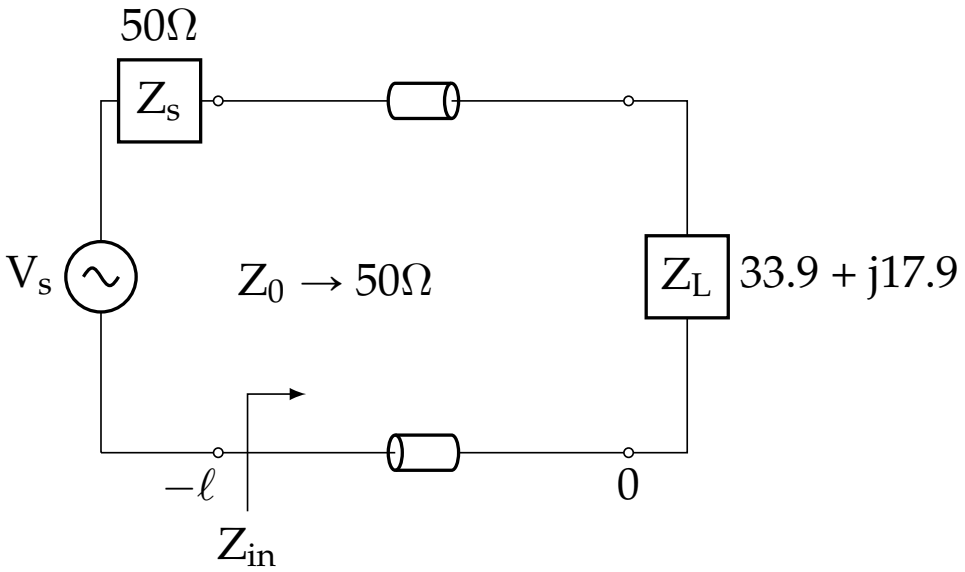
$$\implies Z_C = \frac{1}{j\omega C} \text{ or } Z_L = j\omega L$$

5. Find the length of TX line to make:

$$\implies z_{\text{in}} = 1 + jX = \frac{z_L + j\tan(\beta\ell)}{1 + jz_L\tan(\beta\ell)}$$

Impedance Matching: Example

Match the combined load (Z_L and TX line) to Z_s.



1. Normalize Impedances

$$z_L = \frac{33.9 + j17.6\Omega}{50\Omega} = 0.678 + j0.352$$

2. Find reflection coefficient

$$\Gamma_L = \frac{z_L - 1}{z_L + 1} = \frac{-0.322 + j0.352}{1.678 + j0.352}$$
$$= -0.141 + j0.239 = 0.278\angle 120.7^\circ$$

Impedance Matching: Example (cont'd)

3. Find z_{in} & C or L

$$
\begin{aligned}
|\Gamma_L| = 0.278 &= \left|\frac{z_{in}-1}{z_{in}+1}\right| \\
&= \left|\frac{1+jx-1}{1+jx+1}\right| = \left|\frac{jx}{2+jx}\right| \\
&= \frac{\sqrt{0+x^2}}{\sqrt{2^2+x^2}} \\
0.278^2 &= \frac{x^2}{4+x^2} \\
&\Longrightarrow \; x = \pm 0.579 \\
&\Longrightarrow \; X = \pm(0.579)Z_0 = 28.95\Omega \\
-j28.95 &= \frac{1}{j\omega C} \\
+j28.95 &= j\omega L
\end{aligned}
$$

Impedance Matching: Example (cont'd)

4. Determine Tx Line Length ℓ

$$z_{\text{in}} = 1 + j0.579 = \frac{z_L + j\tan(\beta\ell)}{1 + jz_L\tan(\beta\ell)}$$

Expand, cross-multiply, equate real parts and solve for $\beta\ell$...

$$z_{\text{in},1} = 1 + j0.579 \longrightarrow \ell_1 = 0.065\lambda$$

$$z_{\text{in},2is} = 1 - j0.579 \longrightarrow \ell_2 = -0.230\lambda + 0.5\lambda = 0.270\lambda$$

Impedance Matching: Review

Match the combined load (Z_L and TX line) to Z_s at 1GHz.

