ECE343 Spring 2025

Objective 3.23: Transmission Line Analysis



Lesson	Topics	Reading	Notes
1	Course Overview & Introduction	Ch.1	LO 1.1-1.2
Chapter 2: Electric and Magnetic Fields			
2	Fields, Parameters, Field Quantities	Ch. 2	LO 2.1-2.2
Chapter 3: Transmission Lines			
3	Lumped Element Model	3.1-3.4	LO 3.1
4	Telegrapher's Equations	3.5-3.6	LO 3.1
5	Characteristic Impedance & Wave Propagation	3.7-3.9	LO 3.2
6	Coaxial, Reflection Coefficient, VSWR	3.10,3.12-3.14	LO 3.2-3.3
7	Terminated Transmission Lines	3.15,3.17-3.19	LO 3.4
8	Impedance Matching	3.21-3.23	LO 3.4
9	Impedance Matching	3.21-3.23	LO 3.4
10	Smith Charts	Supplemental	LO 3.5
11	Smith Charts	Supplemental	LO 3.5
12	Smith Charts	Supplemental	LO 3.5
13	Smith Charts	Supplemental	LO 3.5
14	Transients	Supplemental	LO 3.6
15	Reflection/Transmission Coefficient Lab	Lab Packet	LO 3.7
Chapter 4: Vector Analysis			
16	Vector & Coordinate System Review	Ch. 4	LO 4.1-4.4 **GR 1 due**
Chapter 5: Electrostatics			
17	Coulomb's Law	5.1-5.3	LO 5.1
18	Surface and Volume Charge	5.4	LO 5.1
19	Gauss' Law (Integral Form)	5.5-5.6	LO 5.2
20	Gauss' Law (Differential Form)	5.7-5.8	LO 5.2
21	Electric Potential	5.9-5.10, 5.15	LO 5.3

Ch 3 Learning Objectives

3.1 I can describe the origin of the transmission line equation, understand the method of developing a solution, and interpret the solution.

3.2 I can explain the physical meaning of and calculate the following quantities: propagation constant, phase constant, attenuation constant, characteristic impedance, VSWR, reflection coefficient, and transmission coefficient.

3.3 I can use the transmission line equation to calculate the voltage/current at an arbitrary point on a specified transmission line.

3.4 I can use the transmission line equation to match a load to a source using a length of transmission line.

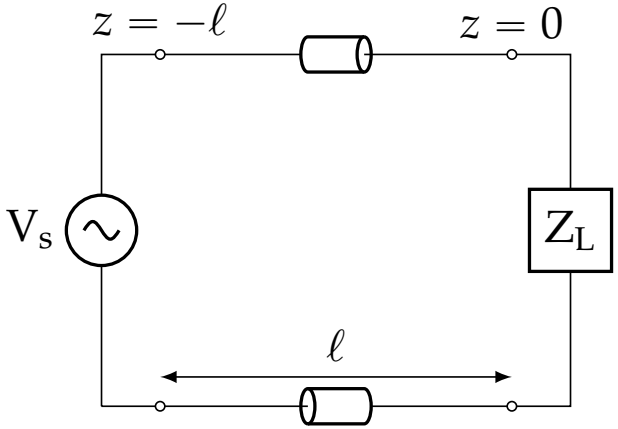
3.5 I can identify the various quantities represented on the Smith Chart.

3.6 I can use a Smith Chart to design a matching network for a given source and load, using series components, parallel components, shorted stubs, and open stubs.

3.7 I can plot the transient response of a load connected to a source through a transmission line.

3.8 I can measure the reflection coefficient across a range of frequencies and interpret the measurement using a network analyzer.

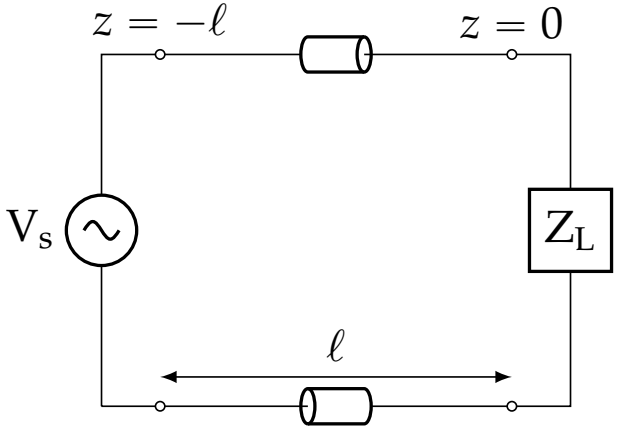
Impedances



Impedances

$$Z_L = Z(z=0) = \frac{V(z=0)}{I(z=0)} = \frac{V_0^+ e^{-\gamma(0)} + V_0^- e^{\gamma 0}}{I_0^+ e^{-\gamma(0)} + I_0^- e^{\gamma 0}} = \frac{V_0^+ + V_0^-}{I_0^+ + I_0^-}$$

Impedances

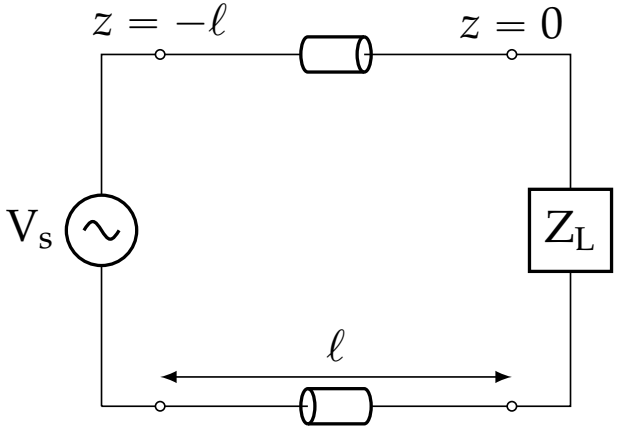


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Impedances



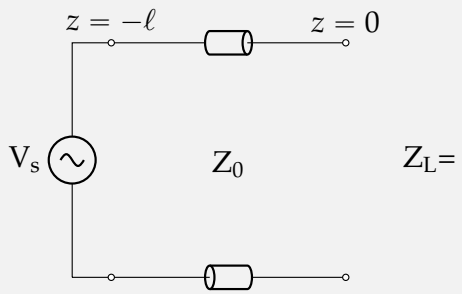
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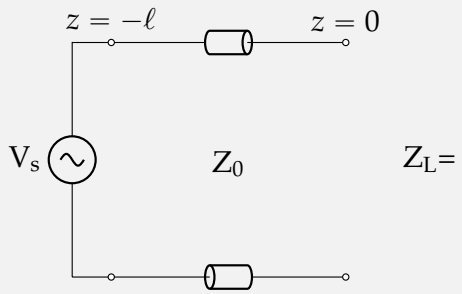
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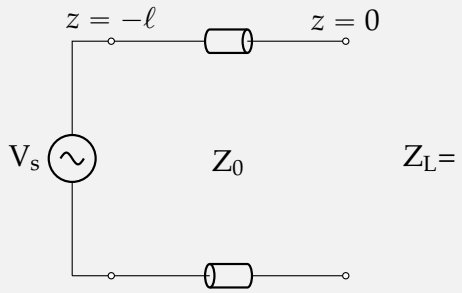
$$\vdots$$

$$\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \leftarrow \text{Reflection Coefficient}$$

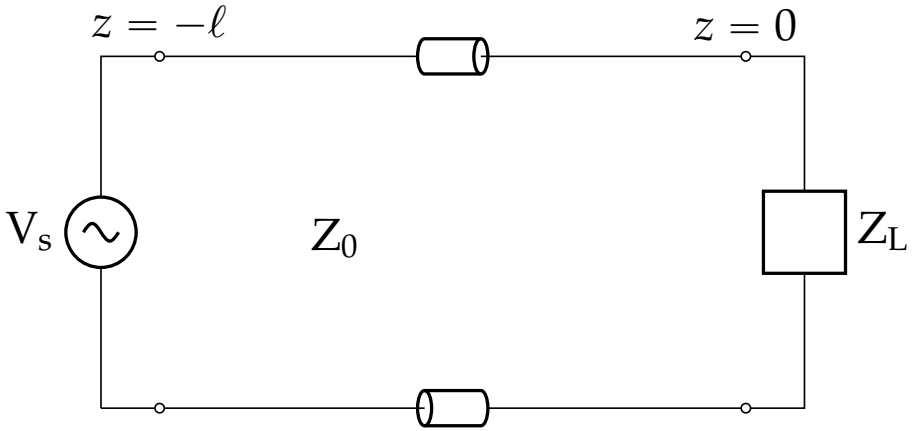
Special Cases for Γ





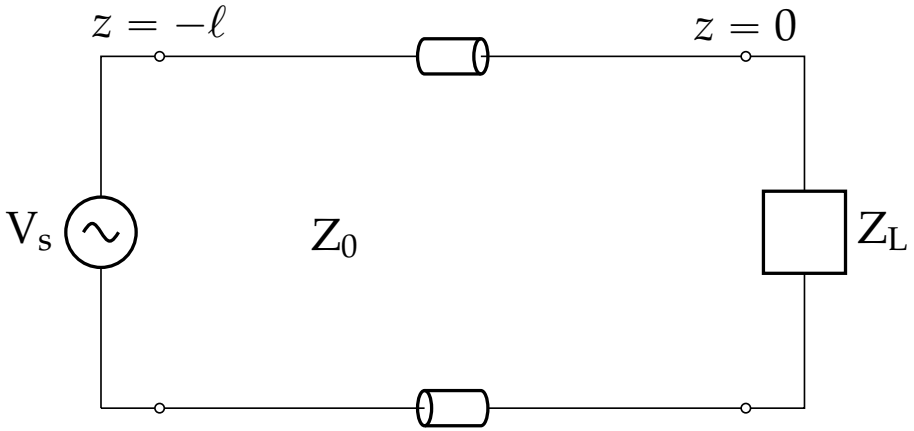


Towards a General Tx Line Equation



Recall $\longrightarrow \tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

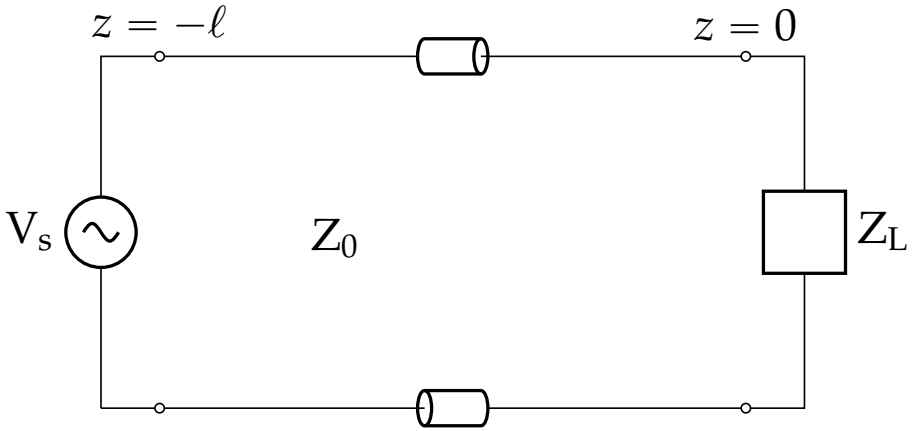
Towards a General Tx Line Equation



$$\text{Recall} \longrightarrow \widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\Gamma(z) = \frac{V_0^- e^{-\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^-}{V_0^+} e^{2\gamma z}$$

Towards a General Tx Line Equation

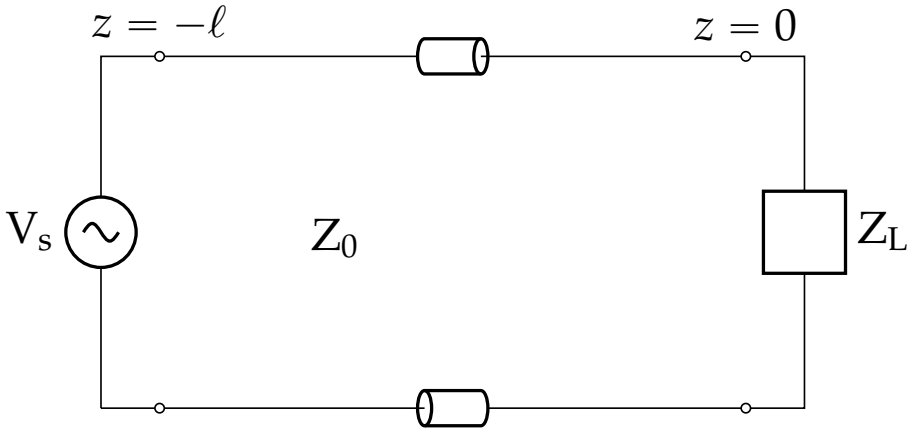


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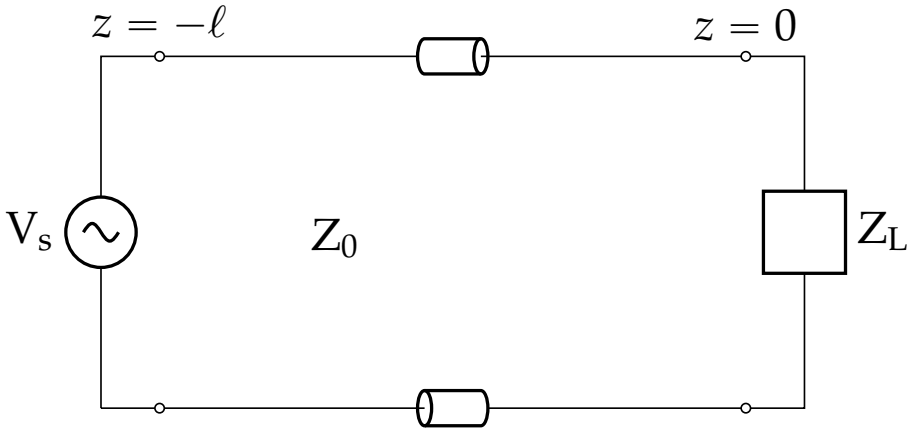
$\Gamma(z) =$ _____

Lossless $\longrightarrow \gamma =$ _____

We can also write:

$$\widetilde{V} = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

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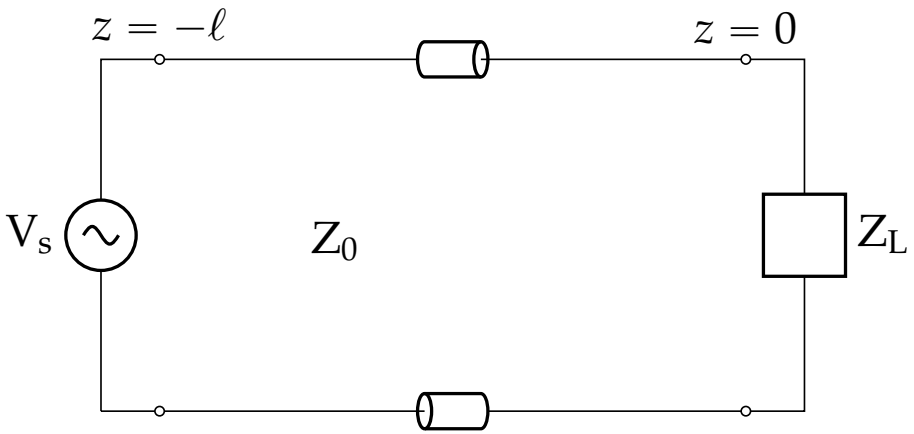
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$$\begin{aligned}\widetilde{V} &= V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ &= V_0^+ e^{-\gamma z} + V_0^+ \Gamma_L e^{\gamma z}\end{aligned}$$

Towards a General Tx Line Equation



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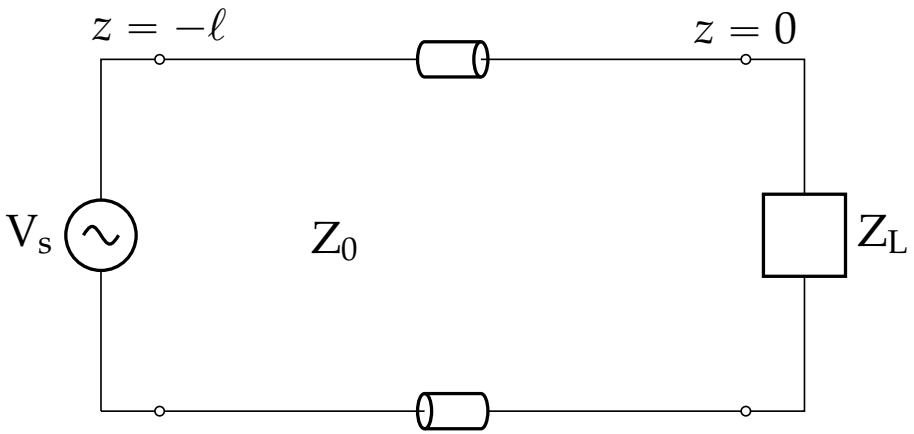
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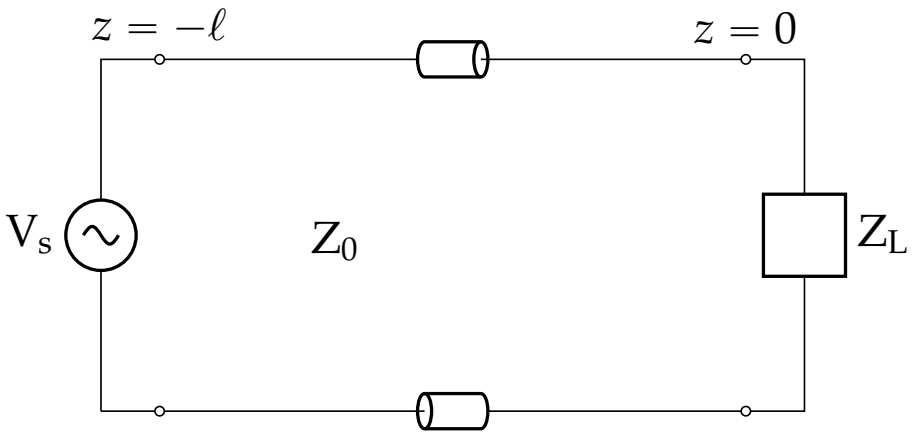
$\widetilde{V} =$ _____

Voltage Standing Wave Ratio

$$\text{SWR} = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

$$|\Gamma| = \frac{\text{SWR} - 1}{\text{SWR} + 1}$$

SUMMARY



Wave Equations

$$\begin{aligned} V(z,t) &= V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \\ &= V_0^+ e^{-\alpha z}\cos(\omega t - \beta z) \\ &\quad + V_0^- e^{\alpha z}\cos(\omega t + \beta z) \end{aligned}$$

$$v_p = \frac{\omega}{\beta} \quad \lambda = \frac{2\pi}{\beta}$$

Summary

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \Gamma(z) = |\Gamma_L| e^{2\gamma z}$$

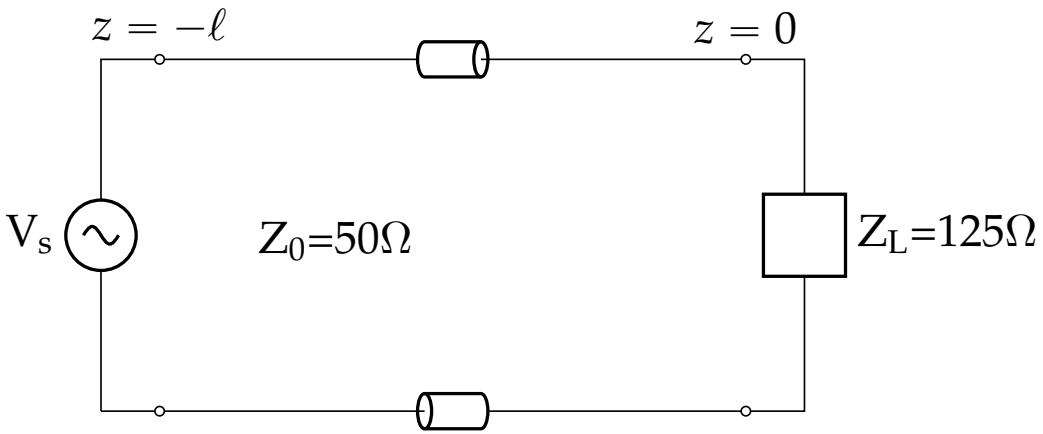
$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \qquad \text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

Examples

Example #1

A radio transmitter is connected to an antenna by a lossless coaxial cable having a characteristic impedance of 75Ω . The coaxial cable is perfectly matched to the transmitter. The input impedance of the antenna is 500Ω . If the peak voltage from the output of the transmitter is $30V$, what is the peak voltage of the reflected wave at the output of the transmitter.

EXAMPLE



- Find the reflection coefficient.
- Find the VSWR.