ECE343 Spring 2025

Objective 1: Transmission Line Equation



Lesson	Topics	Reading	Notes
1	Course Overview & Introduction	Ch.1	LO 1.1-1.2
Chapter 2: Electric and Magnetic Fields			
2	Fields, Parameters, Field Quantities	Ch. 2	LO 2.1-2.2
Chapter 3: Transmission Lines			
3	Lumped Element Model	3.1-3.4	LO 3.1
4	Telegrapher's Equations	3.5-3.6	LO 3.1
5	Characteristic Impedance & Wave Propagation	3.7-3.9	LO 3.2
6	Coaxial, Reflection Coefficient, VSWR	3.10,3.12-3.14	LO 3.2-3.3
7	Terminated Transmission Lines	3.15,3.17-3.19	LO 3.4
8	Impedance Matching	3.21-3.23	LO 3.4
9	Impedance Matching	3.21-3.23	LO 3.4
10	Smith Charts	Supplemental	LO 3.5
11	Smith Charts	Supplemental	LO 3.5
12	Smith Charts	Supplemental	LO 3.5
13	Smith Charts	Supplemental	LO 3.5
14	Transients	Supplemental	LO 3.6
15	Reflection/Transmission Coefficient Lab	Lab Packet	LO 3.7
Chapter 4: Vector Analysis			
16	Vector & Coordinate System Review	Ch. 4	LO 4.1-4.4 **GR 1 due**
Chapter 5: Electrostatics			
17	Coulomb's Law	5.1-5.3	LO 5.1
18	Surface and Volume Charge	5.4	LO 5.1
19	Gauss' Law (Integral Form)	5.5-5.6	LO 5.2
20	Gauss' Law (Differential Form)	5.7-5.8	LO 5.2
21	Electric Potential	5.9-5.10, 5.15	LO 5.3

Ch 3 Learning Objectives

3.1 I can describe the origin of the transmission line equation, understand the method of developing a solution, and interpret the solution.

3.2 I can explain the physical meaning of and calculate the following quantities: propagation constant, phase constant, attenuation constant, characteristic impedance, VSWR, reflection coefficient, and transmission coefficient.

3.3 I can use the transmission line equation to calculate the voltage/current at an arbitrary point on a specified transmission line.

3.4 I can use the transmission line equation to match a load to a source using a length of transmission line.

3.5 I can identify quantities represented on the Smith Chart.

3.6 I can use a Smith Chart to design a matching network for a given source and load, using series components, parallel components, shorted stubs, and open stubs.

What is a Transmission Line?

Definition

A transmission line is a structure intended to transport electromagnetic signals or power.

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A transmission line is a structure intended to transport electromagnetic signals or power.

Purpose

Transmission lines are designed to support guided waves with controlled impedance, low loss, and a degree of immunity from EMI.

Communication Channels

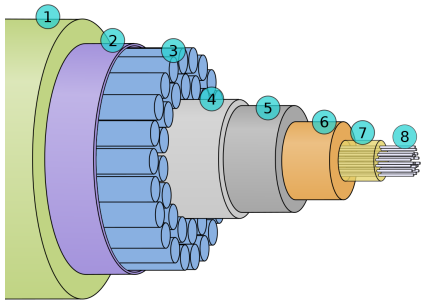
Wired Transmission Lines

Advantages

- Reliable
- Easy to repair
- Relatively secure

Disadvantages

- Wire resistance
- Limited frequency bandwidth



Communication Channels

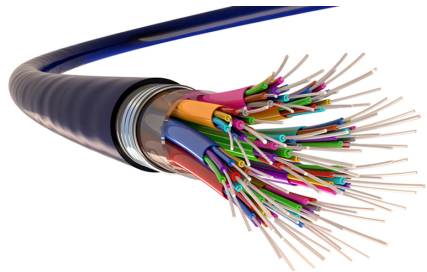
Optical Fibers

Advantages

- Large bandwidth
- Flexible
- Relatively inexpensive
- Low attenuation

Disadvantages

- Difficulty of installation
- Repairability



Communication Channels

Waveguides

Advantages

- Low attenuation
- Precise

Disadvantages

- Heavy/Rigid
- Limited bandwidth
- Difficult to repair
- Expensive



Communication Channels

Wireless

Advantages

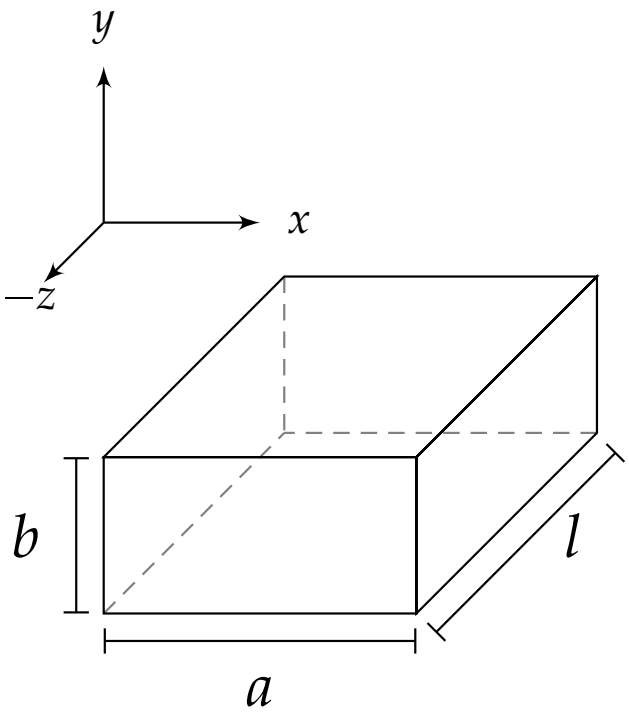
- Very little infrastructure required

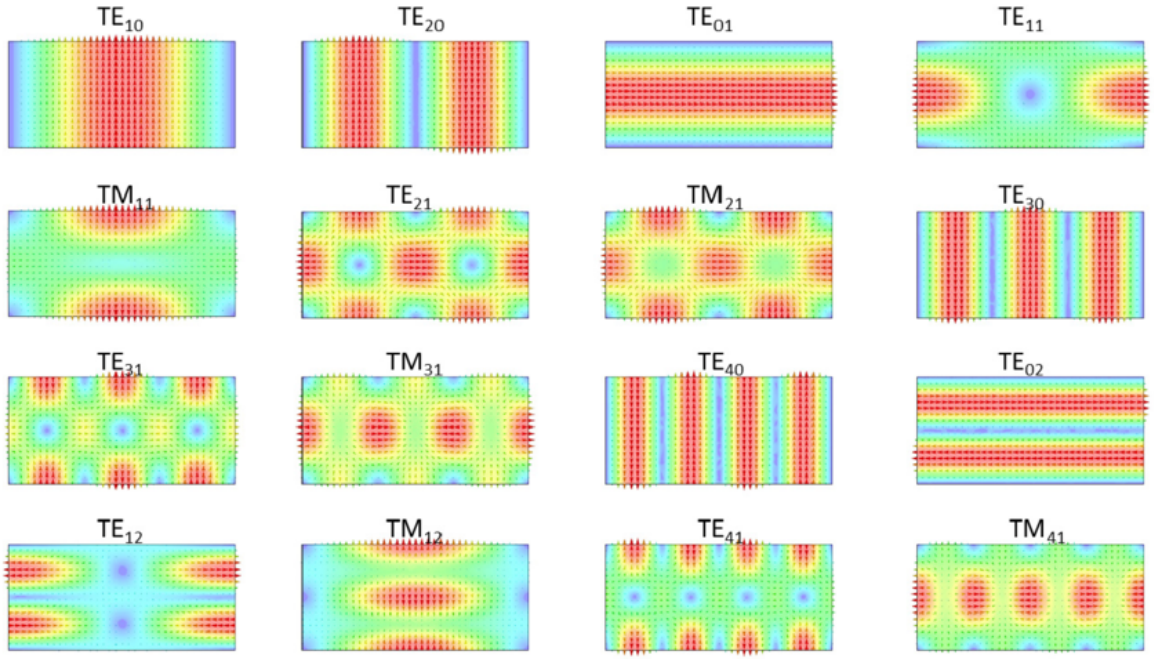
Disadvantages

- Line-of-sight considerations
- Interference
- Security
- Large power required



WAVEGUIDES



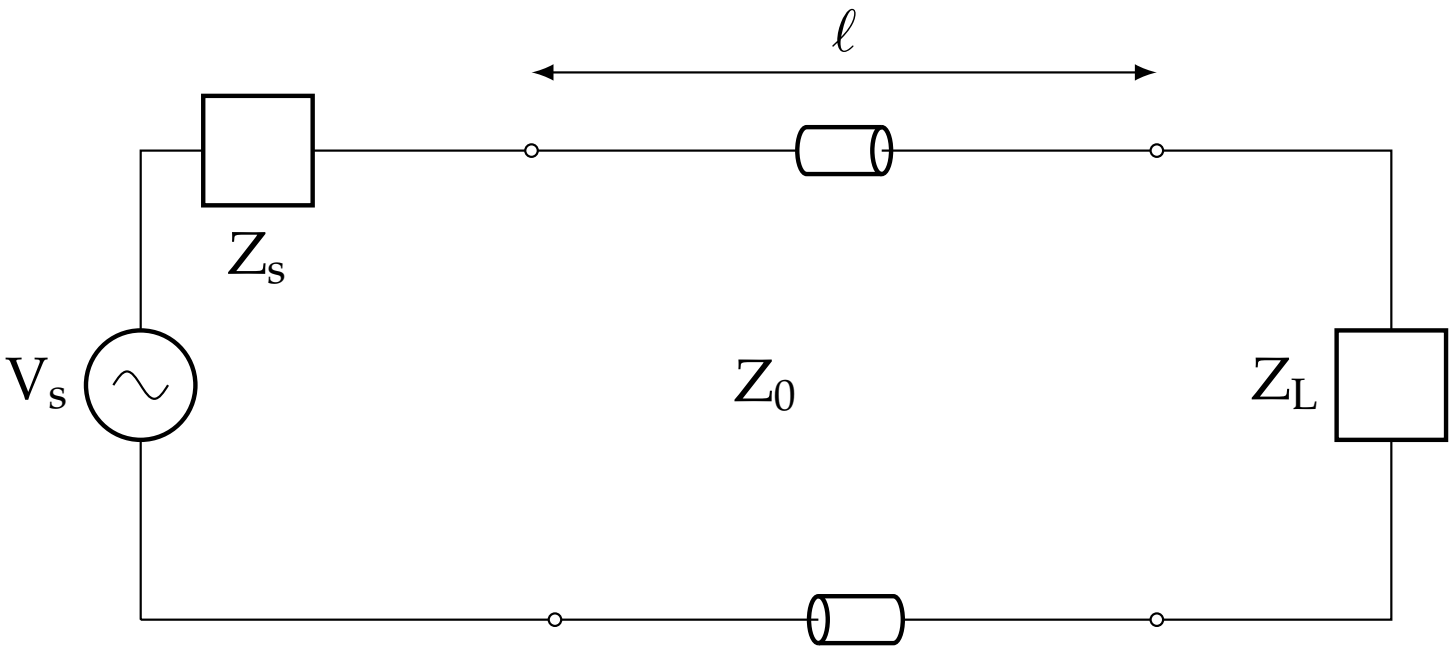


$$\text{cutoff frequency} \longrightarrow f_c = \frac{c}{2a} \longrightarrow \text{HPF!!!}$$

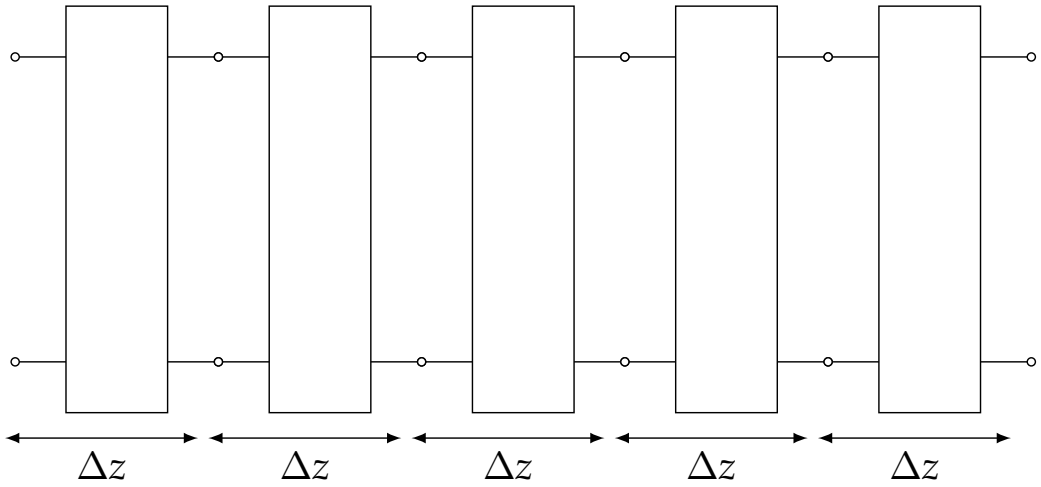
Why Consider Transmission Lines Now?

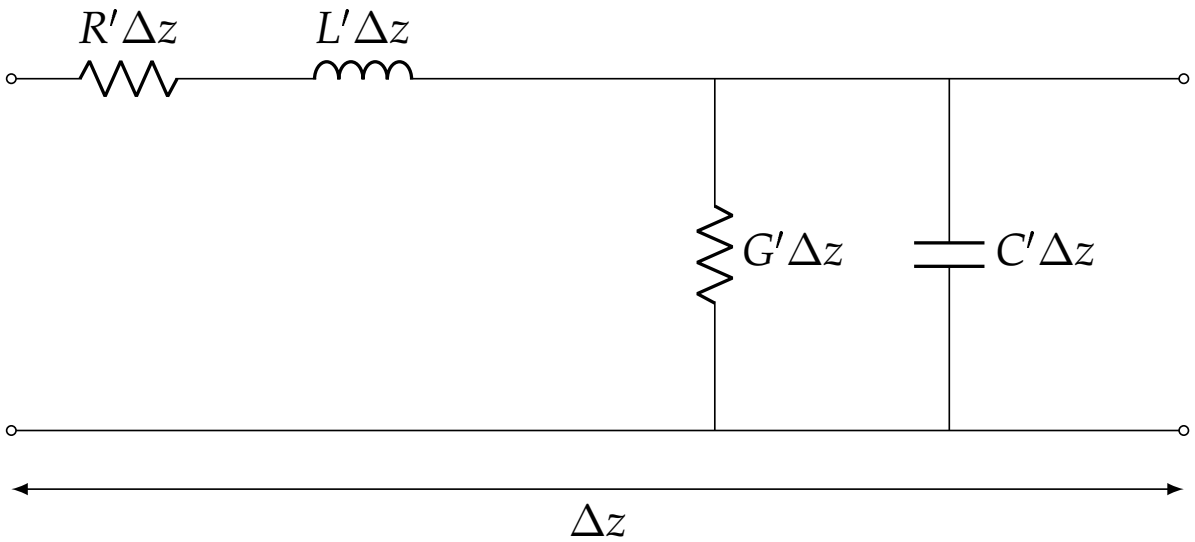
- We use them often, treatment up to this point has been simplified
- When ratio of tx line length (ℓ) to wavelength (λ)
- Wave solutions to transmission line equations
- Reflection, transmission, and interference concepts

Transmission Line Model

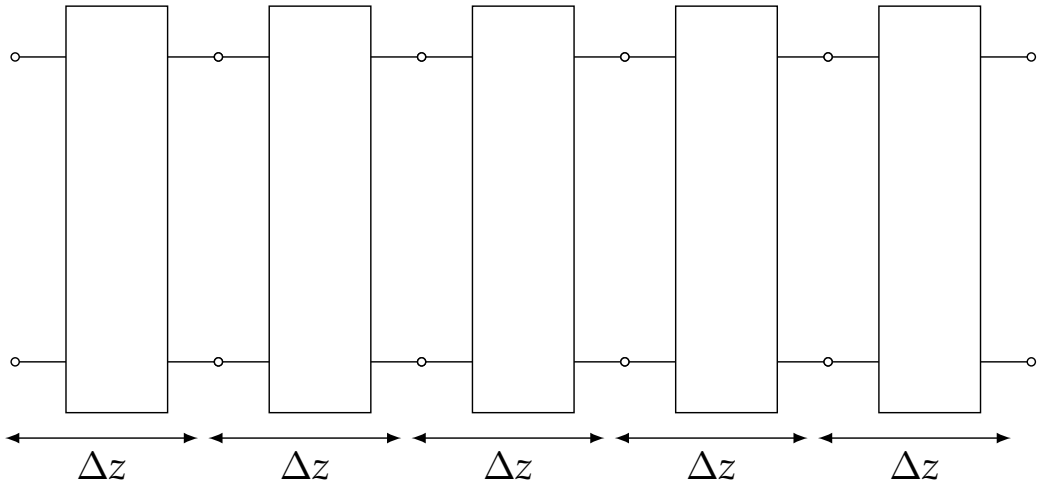


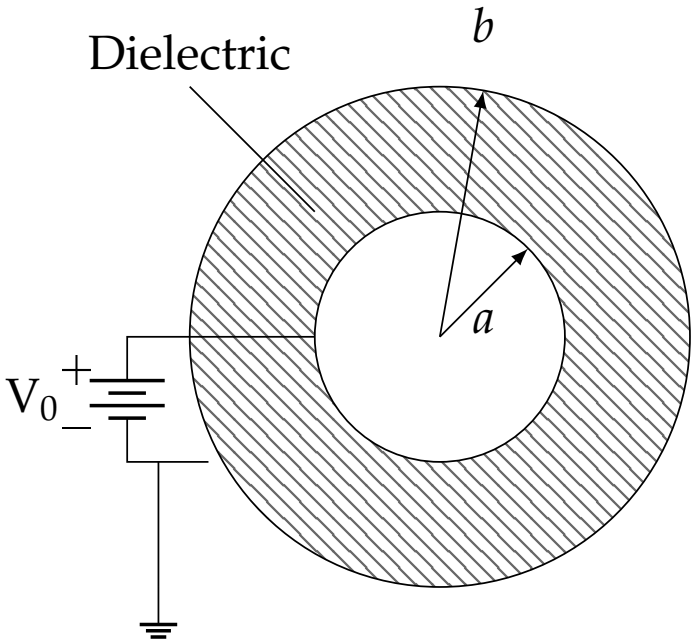
Lumped Element Model

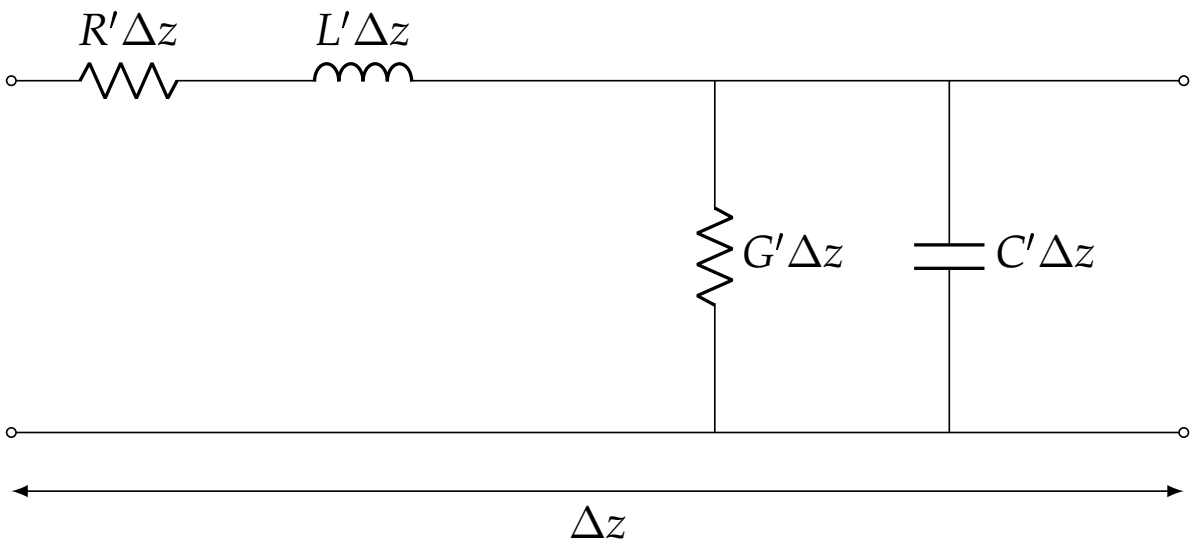




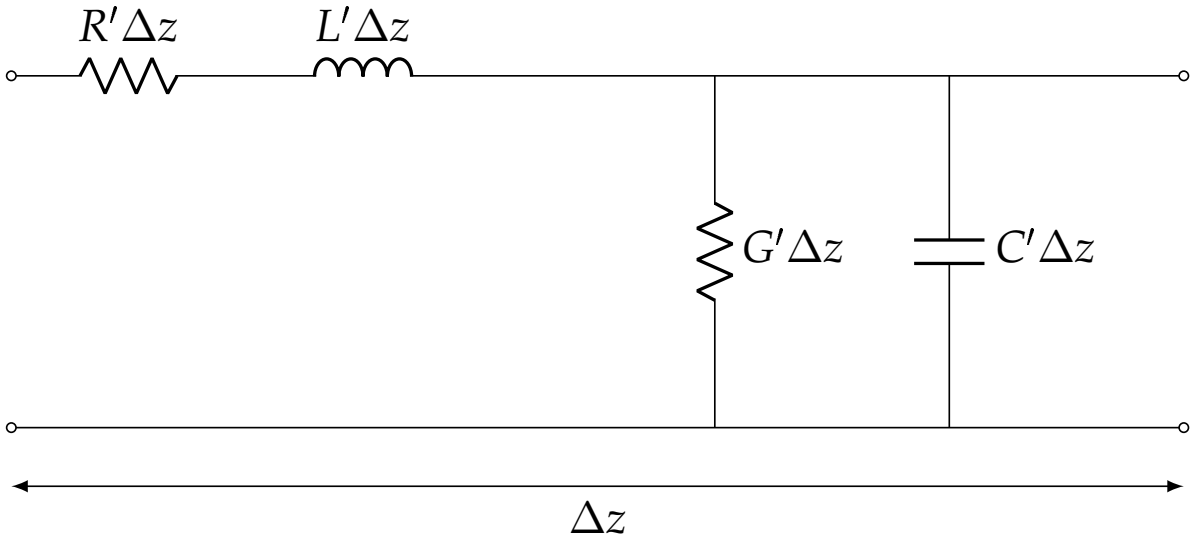
Lumped Element Model







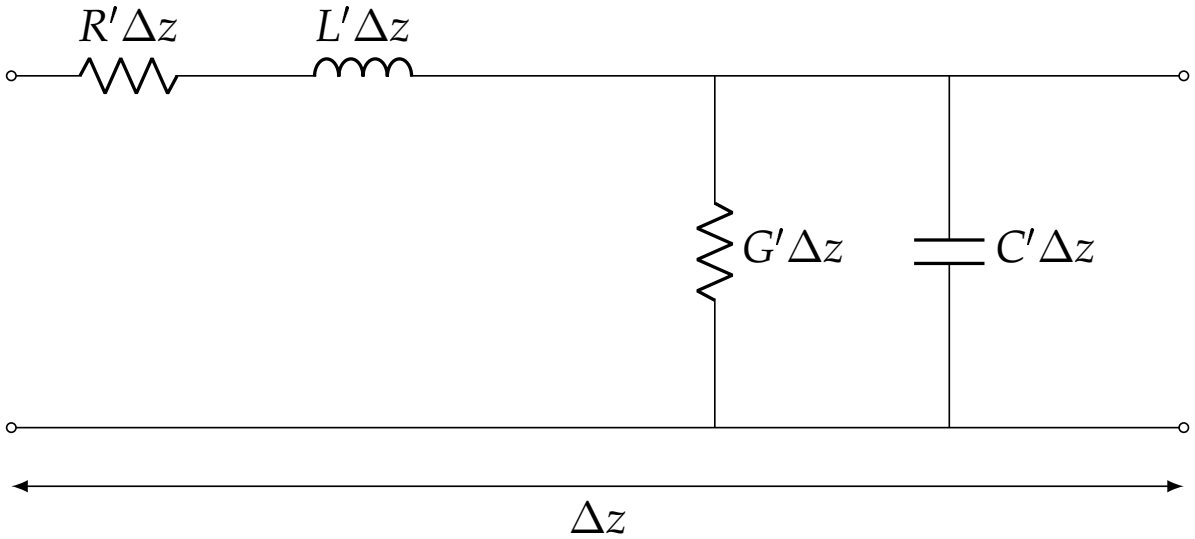
Lumped Element Model: KVL



KVL:

$$-V(z,t) + R'\Delta z i(z,t) + L'\Delta z \frac{\partial i(z,t)}{\partial t} + V(z+\Delta z, t) = 0$$

Lumped Element Model: KVL

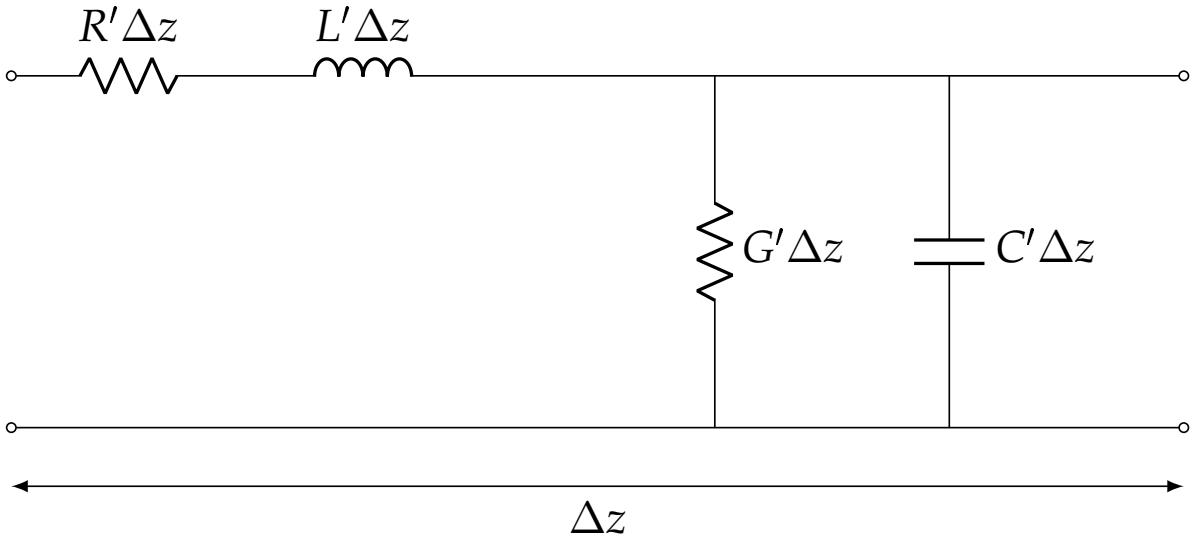


KVL:

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$$\lim_{\Delta z \to 0} \frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = \frac{-R'\Delta z i(z,t) - L'\Delta z \frac{\partial i(z,t)}{\partial t}}{\Delta z}$$

Lumped Element Model: KVL



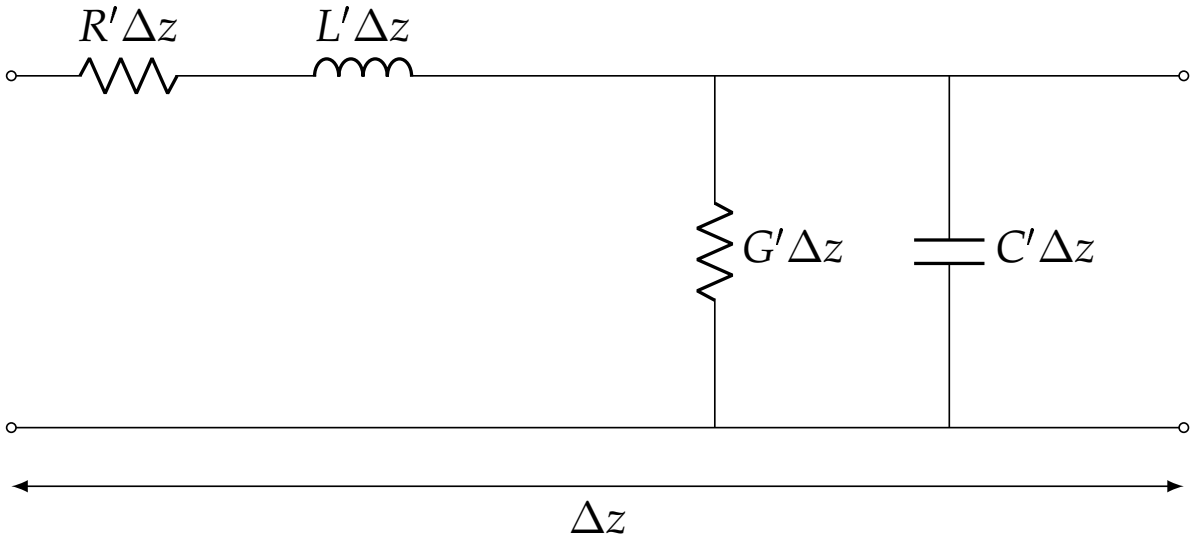
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$$-\frac{\partial V(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$

Lumped Element Model: KCL



KVL:

$$i(z,t) - i(z+\Delta z,t) - G'\Delta z V(z+\Delta z,t) - C'\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t} = 0$$

$$\vdots$$

$$-\frac{\partial i(z,t)}{\partial z} = G'V(z,t) + C'\frac{\partial V(z,t)}{\partial t}$$

Lumped Element Model

Telegrapher's Equations

$$-\frac{\partial V(z,t)}{\partial z} = R'i(z,t) + L'\frac{\partial i(z,t)}{\partial t}$$
$$-\frac{\partial i(z,t)}{\partial z} = G'V(z,t) + C'\frac{\partial V(z,t)}{\partial t}$$

Lumped Element Model

Telegrapher's Equations

$$-\frac{\partial V(z,t)}{\partial z} = R' i(z,t) + L' \frac{\partial i(z,t)}{\partial t}$$
$$-\frac{\partial i(z,t)}{\partial z} = G' V(z,t) + C' \frac{\partial V(z,t)}{\partial t}$$

Phasor form of solutions

$$V(z,t) = \text{Re}\left[\tilde{V}(z)e^{j\omega t}\right] \qquad i(z,t) = \text{Re}\left[\tilde{I}(z)e^{j\omega t}\right]$$

$$\frac{\partial V(z,t)}{\partial t} = \frac{\partial}{\partial t}\text{Re}\left[\tilde{V}(z)e^{j\omega t}\right] \qquad \frac{\partial i(z,t)}{\partial t} = \frac{\partial}{\partial t}\text{Re}\left[\tilde{I}(z)e^{j\omega t}\right]$$

$$= \text{Re}\left[j\omega\tilde{V}(z)e^{j\omega t}\right] \qquad = \text{Re}\left[j\omega\tilde{I}(z)e^{j\omega t}\right]$$

Decouple Telegrapher's Equations

Using Phasor form of solutions:

$$-\frac{\partial \tilde{V}(z)}{\partial z} = R'\tilde{I}(z) + j\omega L'\tilde{I}(z) = \left[R' + j\omega L'\right]\tilde{I}(z) \tag{1}$$

$$-\frac{\partial \tilde{I}(z)}{\partial z} = G'\tilde{V}(z) + j\omega C'\tilde{V}(z) = \left[G' + j\omega C'\right]\tilde{V}(z) \tag{2}$$

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$$-\frac{\partial \tilde{I}(z)}{\partial z} = G'\tilde{V}(z) + j\omega C'\tilde{V}(z) = \left[G' + j\omega C'\right]\tilde{V}(z) \tag{2}$$

Take $\frac{\partial}{\partial z}$ of (1) to get:

$$-\frac{\partial^2}{\partial z^2}\tilde{V}(z) = \left[R' + j\omega L'\right]\frac{\partial \tilde{I}}{\partial z} \tag{3}$$

Decouple Telegrapher's Equations

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$$-\frac{\partial^2}{\partial z^2}\tilde{V}(z) = \left[R' + j\omega L'\right]\frac{\partial \tilde{I}}{\partial z} \tag{3}$$

Now, plug (2) into (3):

$$-\frac{\partial^2}{\partial z^2}\tilde{V}(z) = -\left[R' + j\omega L'\right]\left[G' + j\omega C'\right]\tilde{V}(z)$$

$$\Longrightarrow \quad \frac{\partial^2}{\partial z^2}\tilde{V}(z) - \left[R' + j\omega L'\right]\left[G' + j\omega C'\right]\tilde{V}(z) = 0$$

$$\Longrightarrow \quad \frac{\partial^2}{\partial z^2}\tilde{V}(z) - \gamma^2\tilde{V}(z) = 0 \text{ with: } \gamma = \sqrt{\left[R' + j\omega L'\right]\left[G' + j\omega C'\right]}$$

Lumped Element Model: Wave Equations

Wave Equations

$$\frac{\partial^2}{\partial z^2}\tilde{V}(z) - \gamma^2\tilde{V}(z) = 0$$

$$\frac{\partial^2}{\partial z^2}\tilde{I}(z) - \gamma^2\tilde{I}(z) = 0$$

$$\gamma = \sqrt{[R' + j\omega L'][G' + j\omega C']}$$

Solving Wave Equations

Wave Equations

$$\frac{\partial^2}{\partial z^2}\tilde{V}(z) - \gamma^2\tilde{V}(z) = 0$$

$$\frac{\partial^2}{\partial z^2}\tilde{I}(z) - \gamma^2\tilde{I}(z) = 0$$

$$\gamma = \sqrt{[R' + j\omega L'][G' + j\omega C']} = \alpha + j\beta$$

Solution Method

1. Assume form of solution: _____

Solving Wave Equations

Wave Equations

$$\frac{\partial^2}{\partial z^2}\tilde{V}(z) - \gamma^2\tilde{V}(z) = 0$$

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Solution Method

1. Assume form of solution: _____
2. Find characteristic equation & it's _____

Solving Wave Equations

Wave Equations

$$\frac{\partial^2}{\partial z^2}\tilde{V}(z) - \gamma^2\tilde{V}(z) = 0$$

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Solution Method

1. Assume form of solution: _____
2. Find characteristic equation & it's _____
3. Total solution is the sum of all possible solutions

Solving Wave Equations

Wave Equations

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$$\gamma = \sqrt{[R' + j\omega L'][G' + j\omega C']} = \alpha + j\beta$$

Solution Method

1. Assume form of solution: _____
2. Find characteristic equation & it's _____
3. Total solution is the sum of all possible solutions
4. Enforce _____ & _____

Wave Equation Solutions

Solution Forms

$$\tilde{V}(z) =$$

$$\tilde{I}(z) =$$

$$\gamma =$$

Back to Time Domain

To find the time domain solution, multiply by _____ and _____

$$V(z,t) = \text{Re}\left[\tilde{V}(z)e^{j\omega t}\right] = \text{Re}\left[V_0^+ e^{-\gamma z}e^{j\omega t} + V_0^- e^{\gamma z}e^{j\omega t}\right]$$

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Remember, $\gamma =$ _____, which leads to:

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Remember, $\gamma =$ _____, which leads to:

$$\begin{aligned} V(z,t) &= \text{Re}\left[V_0^+ e^{-\gamma z}e^{j\omega t} + V_0^- e^{\gamma z}e^{j\omega t}\right] \\ &= \text{Re}\left[V_0^+ e^{-(\alpha+j\beta)z}e^{j\omega t} + V_0^- e^{(\alpha+j\beta)z}e^{j\omega t}\right] \\ &= \text{Re}\left[V_0^+ e^{-\alpha z}e^{j(\omega t-\beta z)}V_0^- e^{\alpha z}e^{j(\omega t+\beta z)}\right] \end{aligned}$$

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Instantaneous Solution to Wave Equations

$$V(z,t) = V_0^+ e^{-\alpha z}\cos(\omega t - \beta z) + V_0^- e^{\alpha z}\cos(\omega t + \beta z)$$

Interpretation of Solution

$$V(z,t) = \underbrace{V_0^+ e^{-\alpha z}\cos(\omega t - \beta z)}_{\text{Forward Wave}} + \underbrace{V_0^- e^{\alpha z}\cos(\omega t + \beta z)}_{\text{Reverse Wave}}$$

Interpretation of Solution

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- $V_0^{\pm} \longrightarrow$ Amplitude coefficients

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- $V_0^{\pm} \longrightarrow$ Amplitude coefficients
- $e^{\pm\alpha z} \longrightarrow$ Attenuation term

Interpretation of Solution

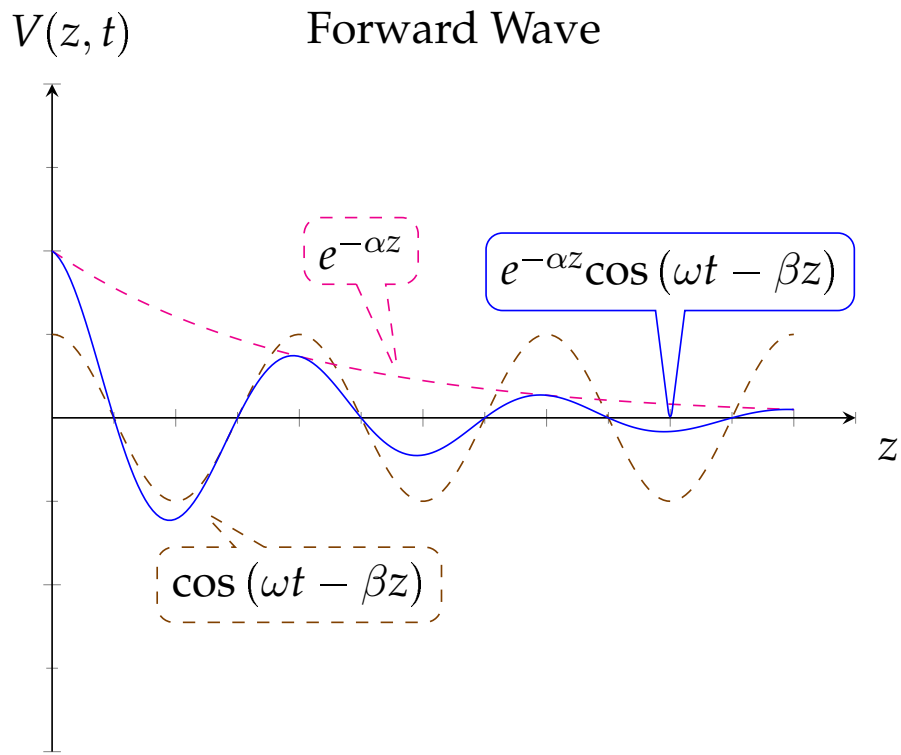
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- $V_0^{\pm} \longrightarrow$ Amplitude coefficients
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- $\cos(\omega t \pm \beta z) \longrightarrow$ Time and space-varying wave

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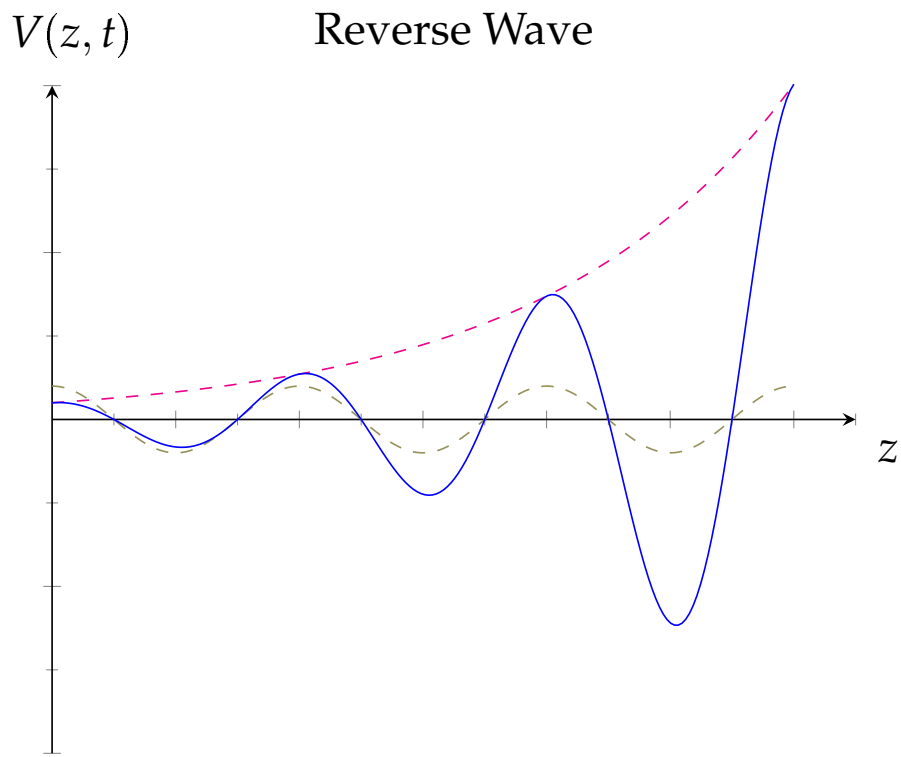
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Interpretation of Solution

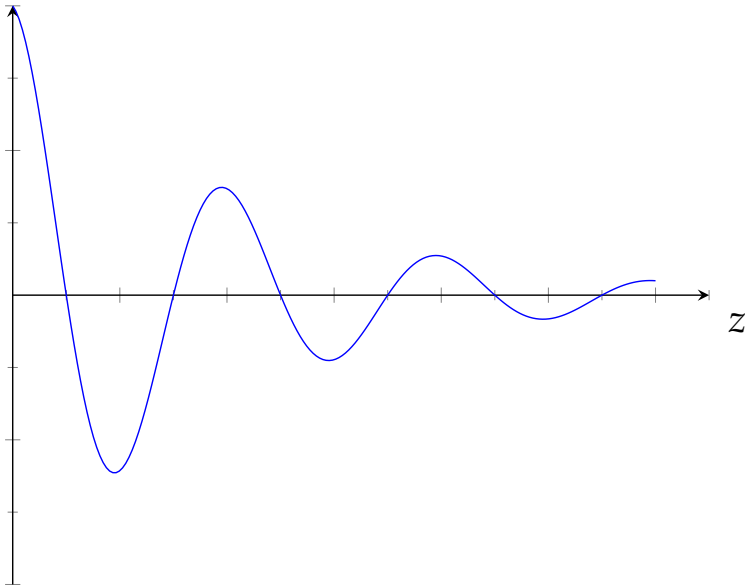
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- $V_0^{\pm} \longrightarrow$ Amplitude coefficients
- $e^{\pm\alpha z} \longrightarrow$ Attenuation term
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Key Parameters of Waves

$$V(z,t) = V_0^+ e^{-\alpha z} \cos(\omega t - \beta z)$$



- Direction = _____
- Wavelength _____
- Speed = _____

Key Parameters for Lossless Case

Lossless/low-loss Conditions

$$\gamma = \sqrt{(R' + j\omega C')(G' + j\omega L')}$$

$R' \longrightarrow G'$ or: $R' << \omega C'$ and $G' << \omega L'$

$$\Longrightarrow \quad \gamma = _____$$

$$\Longrightarrow \quad \alpha = _____ \qquad \beta = _____$$

Coax Cable

$$C' = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} \qquad L' = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$v_p = _____$$

Summary / Review

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$
$$-\frac{\partial \tilde{V}(z)}{\partial z} = \left(R' + j\omega L'\right)\tilde{I}(z)$$
$$-\frac{\partial \tilde{I}(z)}{\partial z} = \left(G' + j\omega C'\right)\tilde{V}(z)$$

Characteristic Impedance - Forward Wave

$$-\frac{\partial \tilde{V}(z)}{\partial z} = \left(R' + j\omega L'\right)\tilde{I}(z)$$

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Characteristic Impedance - Forward Wave

$$-\frac{\partial \tilde{V}(z)}{\partial z} = \left(R' + j\omega L'\right)\tilde{I}(z)$$
$$-\frac{\partial}{\partial z}\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) = \left(R' + j\omega L'\right)\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right)$$

Summary / Review

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Characteristic Impedance - Forward Wave

$$-\frac{\partial \tilde{V}(z)}{\partial z} = \left(R' + j\omega L'\right)\tilde{I}(z)$$
$$-\frac{\partial}{\partial z}\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) = \left(R' + j\omega L'\right)\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right)$$
$$\gamma V_0^+ = \left(R' + j\omega L'\right) I_0^+$$

Summary / Review

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$
$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$
$$-\frac{\partial \tilde{V}(z)}{\partial z} = \left(R' + j\omega L'\right) \tilde{I}(z)$$
$$-\frac{\partial \tilde{I}(z)}{\partial z} = \left(G' + j\omega C'\right) \tilde{V}(z)$$

Characteristic Impedance - Forward Wave

$$-\frac{\partial \tilde{V}(z)}{\partial z} = \left(R' + j\omega L'\right) \tilde{I}(z)$$
$$-\frac{\partial}{\partial z}\left(V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right) = \left(R' + j\omega L'\right)\left(I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}\right)$$
$$\gamma V_0^+ = \left(R' + j\omega L'\right) I_0^+$$
$$Z_0 = \frac{V_0^+}{I_0^+} = \sqrt{\frac{R' + j\omega L'}{\gamma}} = \frac{R' + j\omega L'}{G' + j\omega C'}$$