ECE 343 Equation Sheet - Spring 2025

Department of Electrical and Computer Engineering
United States Air Force Academy

Transmission Lines

$$R' = \frac{1}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)\sqrt{\frac{\pi f \mu}{\sigma_c}} \qquad L' = \frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$G' = \frac{2\pi\sigma_d}{\ln\left(b/a\right)} \qquad C' = \frac{2\pi\epsilon}{\ln\left(b/a\right)}$$

$$\gamma = \sqrt{\left(R'+j\omega L'\right)\left(G'+j\omega C'\right)} = \alpha + j\beta$$

$$\beta = \omega\sqrt{\mu\epsilon} = \frac{2\pi}{\lambda}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \sqrt{\frac{R'+j\omega L'}{G'+j\omega C'}}$$

$$\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{\mathrm{Z_L}-\mathrm{Z_0}}{\mathrm{Z_L}+\mathrm{Z_0}}$$

$$u_p = \frac{\omega}{\beta}$$

$$\mathrm{SWR} = \frac{1+\left|\Gamma_L\right|}{1-\left|\Gamma_L\right|}$$

$$\Gamma(z) = \Gamma_L e^{2\gamma z}$$

$$\widetilde{V}(z) = V_0^+\left(e^{-\gamma z}+\Gamma_L e^{\gamma z}\right)$$

$$\mathrm{Z_{in}} = \mathrm{Z_0}\frac{\mathrm{Z_L}+\mathrm{Z_0}\tanh\left(\gamma\ell\right)}{\mathrm{Z_0}+\mathrm{Z_L}\tanh\left(\gamma\ell\right)}$$

$$\mathrm{Z_{in}} = \mathrm{Z_0}\frac{\mathrm{Z_L}+j\mathrm{Z_0}\tan\left(\beta\ell\right)}{\mathrm{Z_0}+j\mathrm{Z_L}\tan\left(\beta\ell\right)}$$

Statics/BCs/Misc

$$F_{1,2} = \frac{Q_1 Q_2}{4\pi\epsilon R_{12}^3}\vec{R}_{12} \qquad F_{1,2} = \int I_2 d\ell_2 \times B_1$$

$$\vec{E} = \int \frac{dQ}{4\pi\epsilon R^2}\hat{R} \qquad \vec{H} = \int \frac{I d\vec{\ell}\times\hat{R}}{4\pi R^2}$$

$$\vec{E}_{\mathrm{LC}} = \frac{\rho_L}{2\pi\varepsilon\rho}\hat{\rho} \qquad \vec{E}_{\mathrm{SC}} = \frac{\rho_S}{2\varepsilon}\hat{z}$$

$$\vec{H}_{\mathrm{LC}} = \frac{I_{\mathrm{encl}}}{2\pi\rho}\hat{\boldsymbol{\phi}} \qquad \vec{H}_{\mathrm{SC}} = \frac{1}{2}\vec{J}\times\hat{\mathbf{a}}_N$$

$$V = \int \frac{dQ}{4\pi\epsilon r} \qquad \vec{E} = -\nabla V$$

$$d\vec{\ell} = \hat{\mathbf{x}}dx + \hat{\mathbf{y}}dy + \hat{\mathbf{z}}dz$$

$$d\vec{\ell} = \hat{\boldsymbol{\rho}}d\rho + \hat{\boldsymbol{\phi}}\rho d\phi + \hat{\boldsymbol{z}}dz$$

$$d\vec{\ell} = \hat{\boldsymbol{r}}dr + \hat{\boldsymbol{\theta}}r d\theta + \hat{\boldsymbol{\phi}}r\sin\left(\theta\right)d\phi$$

$$\vec{E}_{\tan,1} - \vec{E}_{\tan,2} = 0 \quad \text{or} \quad \hat{\boldsymbol{n}}_{2,1}\times\left(\vec{E}_1 - \vec{E}_2\right) = 0$$

$$\vec{D}_{\mathrm{norm},1} - \vec{D}_{\mathrm{norm},2} = \rho_S \quad \text{or} \quad \hat{\boldsymbol{n}}_{2,1}\cdot\left(\vec{D}_1 - \vec{D}_2\right) = \rho_s$$

$$\vec{H}_{\tan,1} - \vec{H}_{\tan,2} = K \quad \text{or} \quad \hat{\boldsymbol{n}}_{2,1}\times\left(\vec{H}_1 - \vec{H}_2\right) = \vec{K}$$

$$\vec{B}_{\mathrm{norm},1} - \vec{B}_{\mathrm{norm},2} = 0 \quad \text{or} \quad \hat{\boldsymbol{n}}_{2,1}\cdot\left(\vec{B}_1 - \vec{B}_2\right) = 0$$

$$f_{\mathrm{c/o}} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

Fields

$$\nabla^2\hat{E}_s - \gamma^2\hat{E}_s = 0 \qquad \nabla^2\hat{H}_s - \gamma^2\hat{H}_s = 0$$

$$E,H(z,t) = E,H_0^+ e^{-\alpha z}\cos\left(\omega t-\beta z\right) + E,H_0^- e^{\alpha z}\cos\left(\omega t+\beta z\right)$$

$$\gamma = \sqrt{j\omega\mu\left(\sigma+j\omega\epsilon\right)} = \alpha+j\beta \quad \eta = \frac{E_0^+}{H_0^+} = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}}$$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right)}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right)}$$

$$\mathrm{P_{avg}} = \frac{1}{2}\mathrm{Re}\left[\hat{E}_s\times\hat{H}_s^*\right]$$

$$\Gamma_{\mathrm{TE}} = \frac{E_0^r}{E_i^0} = \frac{\eta_2\cos\left(\theta_i\right)-\eta_1\cos\left(\theta_t\right)}{\eta_2\cos\left(\theta_i\right)+\eta_1\cos\left(\theta_t\right)}$$

$$\mathrm{T_{TE}} = \frac{E_0^t}{E_i^0} = \frac{2\eta_2\cos\left(\theta_i\right)}{\eta_2\cos\left(\theta_i\right)+\eta_1\cos\left(\theta_t\right)} = 1+\Gamma$$

$$\Gamma_{\mathrm{TM}} = \frac{E_0^r}{E_i^0} = \frac{\eta_2\cos\left(\theta_t\right)-\eta_1\cos\left(\theta_i\right)}{\eta_2\cos\left(\theta_t\right)+\eta_1\cos\left(\theta_i\right)}$$

$$\mathrm{T_{TM}} = \frac{E_0^t}{E_i^0} = \frac{2\eta_2\cos\left(\theta_i\right)}{\eta_2\cos\left(\theta_t\right)+\eta_1\cos\left(\theta_i\right)} = 1+\Gamma$$

$$\frac{\beta_1}{\beta_2} = \frac{\sin\left(\theta_t\right)}{\sin\left(\theta_i\right)} \qquad \theta_i = \theta_r$$

$$\hat{H}_s = \frac{1}{\eta}\hat{a}_p\times\hat{E}_s \qquad \hat{E}_s = -\eta\hat{a}_p\times\hat{H}_s$$

$$\rho = \rho_0 e^{-\frac{t\sigma}{\alpha}} \qquad \delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

Vector Calculus

$$\nabla V(x,y,z) = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot V(x,y,z) = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$\nabla \times V(x,y,z) = \hat{\mathbf{x}}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \hat{\mathbf{y}}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \hat{\mathbf{z}}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$\nabla V(\rho,\phi,z) = \hat{\boldsymbol{\rho}}\frac{\partial V}{\partial \rho} + \hat{\boldsymbol{\phi}}\frac{1}{\rho}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot V(\rho,\phi,z) = \frac{1}{\rho}\frac{\partial \rho V_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$$

$$\nabla V(r,\theta,\phi) = \hat{r}\frac{\partial V}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{r\sin(\theta)}\frac{\partial V}{\partial \phi}$$

$$\nabla \cdot V(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial r^2 V_r}{\partial r} + \frac{1}{r\sin(\theta)}\frac{\partial V_\theta \sin(\theta)}{\partial \theta} + \frac{1}{r\sin(\theta)}\frac{\partial V_\phi}{\partial \phi}$$

$$\nabla^2 V(x,y,z) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V(\rho,\phi,z) = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V(r,\theta,\phi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin(\theta)^2}\frac{\partial^2 V}{\partial \phi^2}$$

Integrals

$$\int u dv = uv - \int v du$$

$$\int \frac{1}{x}dx = \ln|x| \qquad \int e^{ax}dx = \frac{1}{a}e^{ax}$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x + \sqrt{x^2+a^2}\right)$$

$$\int \frac{1}{(x^2+a^2)^{3/2}}dx = \frac{x}{a^2\sqrt{x^2+a^2}}$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos(bx) + b\sin(bx)\right)$$

$$\int e^{ax}\cos(c+bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin(c+bx) - b\cos(c+bx)\right)$$

Maxwell's Equations

$\iint \epsilon\vec{E}\cdot d\vec{S} = Q_{\text{enc}}$	$\nabla\cdot\vec{E} = \frac{\rho_v}{\epsilon}$
$\oint \vec{H}\cdot d\vec{S} = 0$	$\nabla\cdot\vec{H} = 0$
$\oint_C \vec{E}\cdot d\ell = -\frac{\partial \Psi_{\text{m}}}{\partial t}$ $\oint_C \vec{E}\cdot d\ell = -\iint \frac{\partial \vec{B}}{\partial t}\cdot d\vec{S}$	$\nabla\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t}$ $\nabla\times\hat{\boldsymbol{E}}_s = -j\omega\mu\hat{\boldsymbol{H}}_s$
$\oint_C \vec{H}\cdot d\ell = I_{\text{encl}}$ $\oint_C \vec{H}\cdot d\ell = \int \vec{J}\cdot d\vec{S} + \epsilon\frac{\partial}{\partial t}\iint \vec{E}\cdot d\vec{S}$	$\nabla\times\vec{H} = \sigma\vec{E} + \epsilon\frac{\partial \vec{E}}{\partial t}$ $\nabla\times\hat{\boldsymbol{H}}_s = (\sigma + j\omega\varepsilon)\,\hat{\boldsymbol{E}}_s$

$$\epsilon_0 = 8.854 \times 10^{-12}\text{F/m}$$

$$\mu_0 = 4\pi \times 10^{-7}\text{H/m}$$